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Dissertation

Methodology for the Sizing and Design of Protective Helmets using Three-Dimensional Anthropometric Data

submitted by Michael Gerard Elliott

Department of Environmental Health

In partial fulfillment of the requirements for the

Degree of Doctor of Philosophy

Colorado State University

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WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY MICHAEL GERARD ELLIOTT ENTITLED *METHODOLOGY FOR THE SIZING AND DESIGN OF PROTECTIVE HELMETS USING THREE-DIMENSIONAL ANTHROPOMETRIC DATA* BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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ABSTRACT OF DISSERTATION

METHODOLOGY FOR THE SIZING AND DESIGN OF PROTECTIVE HELMETS USING THREE-DIMENSIONAL ANTHROPOMETRIC DATA

Laser scanning technology for three-dimensional surface digitizing has provided designers of protective equipment and clothing another tool of anthropometric measurement for optimizing fit and improving performance. This technology can provide in a matter of seconds hundreds of thousands of accurate, three-dimensional (3-D) surface coordinates. These 3-D surface points can then be visualized, measured, and manipulated for the design of many devices that require close fitting contact with the human body. One such device is the fighter aircraft pilot helmet. Fighter aircraft pilot helmets, like most high impact protective headgear, must be very close fitting and stable on the head to prevent the helmet from dislodging during impact. It is also desirable to reduce the mass and moments of inertia of these helmets to prevent neck strain

and discomfort while wearing the helmet and reduce the risk of neck injury during an accident.

Until recently, close-fitting protective helmets have been designed using traditional anthropometry and univariate and bivariate statistics of linear measurements of the head. Current designs of fighter aircraft pilot helmets were developed using head length and head breadth. By using only a few measurements of the head to design the helmet, no information on the shape of the human head is employed in the design methodology. Therefore, the remaining dimensions of the helmet are provided from an artistic reconstruction of the head's shape and size.

This dissertation has presented a new method for sizing and design of protective helmets using three-dimensional anthropometric data. The new sizing method includes conducting Principal Component Analysis (PCA) on vectors of the head. The vectors are found by defining a midpoint in the head and then calculating the distance and direction angles from the midpoint to the surface points. The scores from the PCA of the vectors for each of the heads are then used in a K-means clustering routine to partition the sample of heads into different sizes. The helmets are designed for each size based on regression analysis of the vectors and the PCA scores.

Three computer programs were written to determine the midpoint in the head, calculate the vectors of the head, and select the vectors that are of interest for design of the helmets. The first program, named MIDPOINTS, determines

the midpoints of the head based on selected criteria. The program also calculates a number of measurements, such as distance between landmarks and direction angles, using the three-dimensional anthropometric data from laser scanning equipment. The second program, named VECTORS, calculates the distance (i.e., magnitudes) and direction angles of vectors in the head. The starting points for the vectors are the midpoints, determined from the program MIDPOINTS; the ending points for the vectors are the surface points on the head found from the laser scanning equipment. The third program, named VECSELECT, uses the vector data calculated in VECTORS to select a vector of interest. The vector is found by comparing the direction angles of the vector of interest to the direction angles of each of the vectors calculated for a particular head. The vectors that have direction angles closest to the direction angles of interest are then selected and written to an output file for statistical analysis

The dissertation compares the proposed methodology to currently used techniques for the design of close-fitting helmets. One currently used method includes using a bivariate plot of head length and head breadth to select reference heads that are then used for the design dimensions of the new helmet. This research compared the variances of the vectors found from this reference head method to the variances of the vectors found from new method for sizing and design proposed in this dissertation. In most cases, the variances were significantly lower ($p < 0.05$) for the new methodology, than the reference head method. This indicates that helmets designed from the new method would be

better fitting and would have less unused space between the helmet and the head than helmets designed from the reference head method.

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This dissertation can be obtained in an electronic format with full color pictures and drawings by sending an e-mail request to mg_elliott@yahoo.com.

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LIST OF SYMBOLS, ACRONYMS, AND ABBREVIATIONS

3D or 3-D	three-dimensional.
Δ	difference between two variables.
θ	theta; the angle in a spherical coordinate system that is based on the projection of the line of interest onto the XZ plane (see Figure 4-2).
ϕ	phi; is the angle in a spherical coordinate system between the positive Y axis and the line segment of interest (see Figure 4-2).
μ or mu	statistical mean
AAF	Army Air Force
ANOVA	analysis of variance
avgxxxx	name of the average vector for a particular direction. The average is found by adding the two corresponding vectors and dividing by two. The direction angles replace the xxxx.
B-C Arc	Bitragion-Coronal Arc
B-rep	Boundary representation as it relates to 3-D modeling of form.
CAD	Computer-Aided Design

CAM	Computer-Aided Manufacturing
CARD	Computerized Anthropometric Research and Design Labs
CI	Confidence Interval
Circ	Circumference
Const	constant
CSERIAC	Crew Systems Ergonomics Information Analysis Center
cut-off values	values of head length and head breadth proposed for defining helmet sizes; used in reference head method of helmet sizing.
d	Euclidean distance between two points; $d = [(x_s - x_m)^2 + (y_s - y_m)^2 + (z_s - z_m)^2]^{1/2}$ where the subscript s indicates Cartesian coordinate values for a point on the head's surface and the subscript m indicates Cartesian coordinate values of the midpoint.
delta	difference between head length or breadth and the design helmet's length or breadth, respectively, for a particular size helmet.
diffxxxx	The difference between corresponding vectors at the associated direction angles. The xxxx is replaced by the direction angles.
EDMA	Euclidean distance matrix analysis
FESA	Finite-Element Scaling Analysis
FDM	form difference matrix
FOHMD	Fiber Optic Helmet Mounted Display
HMD	Helmet-Mounted Display

MIDPOINTS	Fortran computer program for calculating the midpoint of a head for helmet design.
mm	millimeters
NURBS	non-uniform rational B-splines
p	probability value
PA	Procrustes analysis
PC1 – PC4	First principal component through fourth principal component
PCA	Principal Component Analysis
Q1	first quartile
Q3	third quartile
r or R	correlation coefficient (Pearson's)
r^2, R-sq, or R^2	coefficient of determination
s	standard deviation
vxxxx or vnxxxx	name of a particular vector calculated in the program VECTORS. The xxxx is replaced by the direction angles. The n represents a negative for the θ direction angle.
VECSELECT	Fortran computer program for selecting head vectors from the vectors calculated in the program VECTORS.
VECTORS	Fortran computer program for calculating vectors of the head from laser scanning data.
VR	virtual reality
x, y, and z	Cartesian coordinates

CHAPTER 1

1 INTRODUCTION

This research was an investigation into the development of a methodology for the design and sizing of protective headgear using three-dimensional surface anthropometric data. This introductory chapter has five major parts: background, problem statement, research objective, user needs, and data source. Chapter 2 is a literature review of relevant publications of the design and sizing of protective headgear. Chapter 3 describes the basis for the design and sizing methodology presented in this disseration. Chapter 4 discusses the computer programs developed for the determination of vectors from a midpoint in the head to the surface points. The computer programs are used in the in the design and sizing methodology. Chapter 5 describes the results of the statistical analysis performed on these vectors of the head for design and sizing of protective headgear. Chapter 6 provides a summary and the conclusions of this research.

1.1 Background

The Air Force Computerized Anthropometric Research and Design (CARD) Laboratory conducts anthropometric research for the design of new

helmets and other protective equipment for U.S. Air Force aviators and other military personnel. As part of their research efforts, they began a project in July 1997 to improve the ergonomic aspects of a new Fiber Optic Helmet Mounted Display (FOHMD). The CARD Labs, along with other organizations, are developing this new FOHMD for use by the U.S. Air Force, and other U.S. military services and NATO forces. This study is assessing the comfort, stability, and optic accommodation of the new FOHMD by conducting fit tests, administering questionnaires, and obtaining 3-dimensional (3-D) scanning and traditional anthropometric data. Recommendations for modifications to the new FOHMD equipment will then be proposed by CARD Labs. The recommendations proposed by the CARD Labs will be based on results of fit tests and may include changes in the shapes and sizes of the helmet liners for accommodation of a larger percentage of the user population.

1.2 Problem Statement

Currently, no systematic methodology exists for the design and sizing of flight helmets or helmet liners using the 3-dimensional (3-D) data obtained from head surface scanning equipment. Previous design and sizing methods have used traditional anthropometric measurements and univariate statistics. Traditional anthropometry, which measures linear distances between homologous landmarks, fails to capture the head's curvature and other features of the surface which makes it difficult to design a good fitting helmet or helmet liner. Three-dimensional (3-D) head scanning, which digitizes the surface of the

head in a Cartesian coordinate system, can overcome the problems presented by traditional anthropometry. However, no systematic methodology exists for using this 3-D data for the design and sizing of protective headgear such as flight helmets or helmet liners.

1.3 Research Objective

The objective of this research was to develop a systematic and organized methodology for the design and sizing of protective headgear using the 3-D head anthropometric data obtained from the scanning equipment. Mathematical and statistical methods for characterizing and comparing the 3-D surface scanning data and for providing a parsimonious method for the design and sizing of helmet liners have been investigated. This research project included the following tasks that are summarized below:

1.3.1 Development of Vectors from a Reference Midpoint

To provide a means of summarizing and analyzing the 3-D head anthropometric data, a point of reference was established in the center of each scanned head. Vectors were then calculated from this reference midpoint to points on the surface of the head. The calculation of the vector includes determining the magnitude, that is, the distance from the midpoint to the surface point, and determining the direction angles. The direction angles are determined by using the midpoint as the starting point and the surface point as the ending point of the vector. This method of developing vectors from head surface scans was first described by Bradtmiller and Beecher (1994) for use in developing

artificial head forms for bicycle helmet design. The 1990 U.S. Air Force anthropometric survey, which obtained both traditional and 3-D head surface scanning data for over 300 male rated officers, will be used as the data set for this research effort.

1.3.2 Statistical Clustering of Head Forms

Clustering is the grouping of similar objects into characteristic classes. For headgear design and sizing, groups of similarly shaped and sized heads have been formulated. The headgear is then designed for each specific group. Robinson (1994) describes and uses a clustering technique for the design and sizing of flight oxygen masks based on groups of similarly shaped and sized faces. Two broad categories of clustering are fuzzy and crisp. Fuzzy clustering refers to situations where the same object may belong to more than one cluster. Crisp clustering refers to classical clustering where an object can belong to one and only one group. Both types of clustering were investigated in this research project.

1.3.3 Principal Component Analysis

Principal Component Analysis (PCA) is a multivariate technique in which a number of related variables are transformed to a smaller set of uncorrelated variables. For headgear design and sizing, this means transforming the multitude of 3-D data obtained from head surface scanning to a few variables, which are then used for determining the designs and sizes of the headgear. Specifically, the vectors developed from the head's reference midpoint have

been used as the variables for the PCA. The results of the PCA were then used in the clustering routines for design and sizing of the headgear.

1.3.4 Mathematical Descriptions of the Head's Shape and Size

Many techniques are available for describing geometrical shapes such as the human head. These include mathematical primitives such as quadrics, and more complex models such as nonuniform rational B-splines (NURBS) and Fourier methods. This research project will review these methods and their applicability for design of headgear. Mathematical techniques, which use anatomical landmarks and the linear distances obtained from traditional anthropometry, were also reviewed. However, the major focus of the research was to develop a design and sizing methodology that does not rely on anatomical landmarks or traditional anthropometry.

1.3.5 Multivariate Regression Analysis

Using traditional and 3-D scanning anthropometric data, in combination, were also investigated for the design and sizing of headgear. Specifically, regression of traditional anthropometric measurements as independent variables and scanning data as the dependent variable was explored. Groups of heads were formulated into sizes based on the traditional anthropometry. These groups were then used in regression models for prediction of the 3-D surface measurements for the design of the protective headgear.

1.4 User Needs

The methodology developed in this research can be used to formulate the design and sizes of protective headgear based on the 3-D head surface data. Designers of flight helmets can use this methodology when developing the new shapes and sizes of prototype helmets. Designers of other types of protective headgear, such as construction hardhats, bicycle helmets, motorcycle helmets, skiing helmets, and other recreational sport protective helmets, can use this methodology as well. The methodology will maximize the number of individuals who have protective headgear that fits, when only a small number of predetermined sizes are available (usually small, medium, and large). The methodology could also be applied to other body parts, not just the head, requiring close fitting protection with a minimal number of sizes.

For protective helmets, fit is critical for performance. The purpose of protective headgear is to prevent injury to the head by the absorbing energy from an external force. For example, flight helmets are designed to prevent head injury from shrapnel or other debris during ejection. Army combat helmets are designed to prevent injury from bullets or other combat-related projectiles. Bicycle helmets are designed to prevent head injury during a fall or accident. The materials used in these protective helmets is based on what the helmet is expected to do and what magnitude of force the helmet is required to absorb and prevent injury to the wearer.

For all of these helmets, a good fit could be the difference between a fatal injury and no injury at all. For example, protective helmets that are too loose may become displaced from the wearer's head during an accident resulting in the helmet not covering the head properly to prevent injury. Conversely, protective headgear that is too tight will be uncomfortable and may cause the wearer to take the helmet off, position the helmet incorrectly on the head, or cause an unnecessary distraction resulting in the reduction in human performance. This last statement is critical for jobs requiring a great degree of concentration where performance is critical. Operation of a fighter aircraft during wartime would be an example where a poor fitting helmet would be extremely undesirable.

Another reason why the users of flight helmets need headgear that fit to the highest degree possible is that the use of helmet-mounted displays (HMDs) requires helmets that are highly stable. Any shift of the helmet on the head or unnecessary movement could prevent proper viewing of the HMD. Helmet-mounted displays are becoming increasingly popular in use throughout military and civilian activities. Medical operations, such as endoscopic surgery, and robotic applications, such as for bomb disposal, are just a few examples of the use of HMDs, and where a poor fitting helmet could reduce human performance resulting in disaster.

1.5 Data Source

Three-dimensional and traditional anthropometric data from the 1990 U.S. Air Force anthropometry survey were used in this research effort. This anthropometric survey included collection of head surface scans and traditional anthropometry from Air Force aviators throughout the United States. This dissertation was aimed at development of a design and sizing methodology, rather than in the specific application of anthropometric data for the design of a helmet system. The methodology developed in this dissertation can be applied to any anthropometric data set and the use of the 1990 U.S Air Force Anthropometric survey was an example of the application of the methodology. The 1990 U.S. Air Force survey provides only a small sample of the helmet wearing population in the Air Force. Also, this helmet wearing population is changing demographically as more women and minorities enter the Air Force and become aviators. It is recommended that before any new designs for helmets are developed that up-to-date anthropometric information be collected on the present day population of Air Force helmet wearers. This new anthropometric set could then be used along with the methodology described in this dissertation for the design and sizing of a new helmet.

CHAPTER 2

2 LITERATURE REVIEW

This chapter provides a review of the literature related to the methodologies for the design and sizing of flight helmets. The chapter is divided into four sections. The first section provides a discussion of anthropometric measuring techniques including both traditional and 3-D surface anthropometry. The second section reviews the mathematical methods for describing the head's shape and size, which are needed for design and sizing of protective headgear. The third section explores the statistical and mathematical techniques for comparing form including methods that rely on anatomical landmarks and those that do not. The last section discusses the history of flight helmet design in areas that concern fit and accommodation.

2.1 Anthropometric Measuring Techniques

Anthropometry (from the Greek anthropos, "human," and metron, "measure") is the biological science of measuring the size, weight, and proportions of the human body (Farkas, 1994, p. 17). Anthropometric measuring techniques can be divided into two groups: traditional and 3-D. A brief discussion of each of these groups is provided below.

2.1.1 Traditional Anthropometry

Traditional anthropometry is defined as "anthropometric measurements using traditional tools including calipers, tape measures, anthropometers and measuring fixtures" (Robinette et al. eds., In-press, p. 175.). Traditional anthropometric data is collected using homologous body surface points (for example, anatomical landmarks with biological correspondence from subject to subject). Figure 2-1 shows a few traditional anthropometric instruments including a sliding caliper, a spreading caliper, and a tape measure.

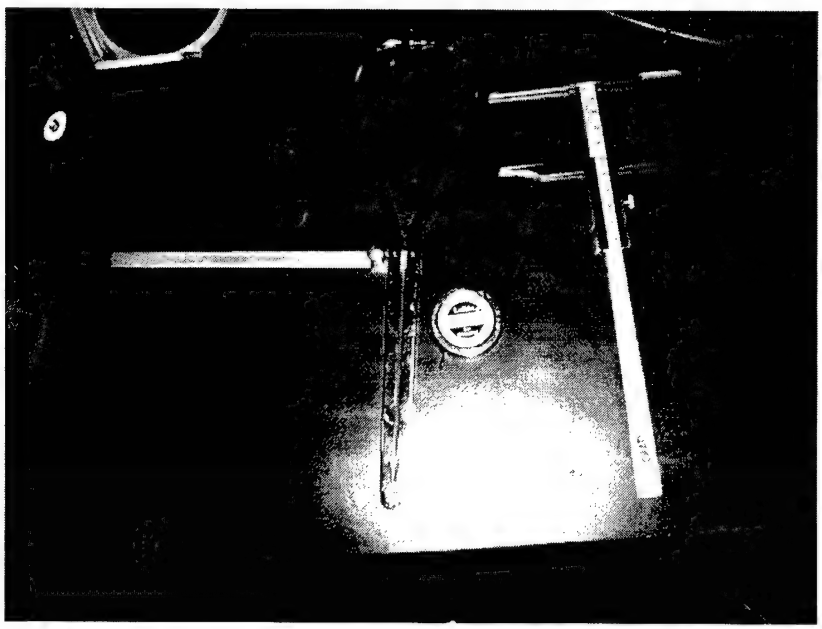


Figure 2-1. Traditional Anthropometric Instruments. From left to right: spreading caliper, soft tape, and sliding caliper.

These instruments are used in helmet fit testing and head anthropometric studies conducted at the CARD Laboratories (see Figure 2-2). The sliding caliper is used to measure the linear distances between two landmarks in the same plane or in neighboring planes. The spreading caliper measures the projective linear

distances between distant surfaces and various planes. The tape measure is used for measuring the tangential linear distances taken along the skin surfaces.

Figure 2-2. Traditional Anthropometric Measurement of the Head at CARD Labs for Helmet Accommodation Study. Soft tape is used to measure the supraorbital arc.

Throughout this century, traditional anthropometry has been used in a multitude of design applications including automobiles, aircraft, and clothing, to name a few. A large number of military and civilian traditional anthropometric surveys have been completed and much of this information is available in data repositories. The Crew Systems Ergonomics Information Analysis Center (CSERIAC) maintains a data repository of over 30 anthropometric surveys completed in many parts of the world and on many different groups of individuals. A similar data repository called ErgoDataOne, maintained in France at the Universite Rene Decartes in Paris, is also available.

2.1.2 Three-dimensional (3-D) Surface Anthropometry

Three-dimensional surface anthropometry is defined as "an extension of anthropometry to the study of the 3-D geometric definition of both external and internal human body tissue boundaries" (Robinette et al. eds., In-press, p. 175). While traditional anthropometry is the study and technique of human body measurement, 3-D surface anthropometry extends the study of the human body to 3-D geometry and morphology of external body tissues (Robinette, In-press, p. 1). It includes the acquisition, indexing, transmission, archiving, retrieval, and analysis of body surfaces and their variability (Robinette, In-press, p. 1).

One method of obtaining 3-D surface measurements of the human body is stereophotogrammetry. Hertzberg et al. (1957), Herron (1972), and Coblentz (1991) were some of the more recent researchers in the use of this technology for capturing biological form. The term biostereometrics was originally coined by Herron to describe the science of measuring and describing, in mathematical terms, the 3-D shape and size of biological objects (Herron, 1972). Stereophotogrammetry captures the exterior surface with linked pairs of photographs. Good reviews of stereophotogrammetry, including a history of its development, theoretical concepts, and its variety of applications, are provided by Herron (1972) and Karara (1989).

Other techniques for obtaining and measuring the 3-D shape and size of objects, which fall into the science of biostereometrics, include moulages,

holography, mechanical digitizers, and structured-light surface scanning. Altobelli (1994) and Rioux and Bruckart (In-press) provide detailed descriptions of moulages, holography, and mechanical digitizers for use in biostereometrics. The 3-D anthropometric data used in this dissertation are from structured-light surface scanning taken with a Cyberware Digitizer and a brief outline of this technology follows.

2.1.2.1 Cyberware Digitizer

Three-dimensional surface digitization, such as from structured light surface scanning, is a technology that relies on the precise acquisition of topographic data in a digital format that is interpreted by computer systems. The Cyberware Digitizer uses a structured light technique to provide an extremely accurate and fast means of three-dimensional head surface digitization. Cyberware Digitizers are entirely noninvasive and provide non-contact acquisition of three-dimensional data with no distortion of the surface tissue being measured.

The Cyberware Digitizer 4020 RGB/PS-D is the current model used by the CARD Laboratories for collection of 3-D anthropometric data (see Figure 2-3). This surface digitizer is based on the application of triangulation geometry to determine range data. This Cyberware Digitizer uses a helium-neon laser that is diverged through stationary optics into a vertical plane of light. This vertical laser light forms a stripe that illuminates the subject being scanned. This vertical light stripe completely traverses the surface of the subject in a rotational manner.

After reflecting off the subject, the light passes through a series of mirrors and prisms to two cameras. One camera is used to record the range to

Figure 2-3. Cyberware Laser Digitizer used at CARD Labs. Notice skull cap is worn to compress hair for helmet accommodation study.

subject data and the other camera is used to record the color. Since the reflected light is viewed obliquely by the camera recording the range data, the stripe changes shape dependent on the distance from the camera (see Farkas, 1994, Figure p. 225, for schematic representation). To make the mathematical calculation of the range, the path of light forms a triangle with a fixed probing

angle at the point of surface contact and a variable deflection of reflected light in the camera's field of view that is a function of depth information. The range and color data are recorded at 512 regular intervals in the vertical direction while the cameras and light stripe circle the subject being scanned. Changes in the vertical stripe's shape are recorded in 512 equally spaced successive images during rotation to determine the 3-D shape and size of the subject's head being scanned. Therefore, the resulting data set consists of approximately 512 x 512 (that is, 262,144) range and color points of the head. The technique has been shown previously to be highly accurate in the surface measurement of three-dimensional objects (Bush et al., 1996, Deacon, et al., 1991; Motoyoshi, et al., 1992). Those interested in obtaining more details on the Cyberware 3-D Digitizer 4020 RGB/PS-D, including its functionality and technical specifications, will find useful information in Hoffmeister et al. (1996).

2.1.3 Advantages of 3-D Surface Anthropometry

Robinette (In-press) provides the most recent discussion of the advantages of 3-D surface anthropometry when compared to traditional methods. She first describes the fact that traditional anthropometric measurements can not be used to reconstruct an object's shape and size without some interpolation of curves or artistic interpretation. For example, if a complete set of traditional anthropometric data were provided of the head, and one was asked to reconstruct the head, filling in the missing details left from the traditional methods will lead to a large number of ambiguous contours to

represent the head's shape and size. This limitation has been identified by a number of author's including Herron (1972, p. 80), Lele (1991), Richtmeier et al. (1992, p. 284), and Altobelli (1994, p. 220).

The second limitation of traditional anthropometry discussed by Robinette (In-press) is that many traditional measurements are dependent entirely upon the orientation of the body segment. She provides an example of the problem caused by this limitation using the measurement of tragion (near the ear) to vertex (top of the head) when used for designing flight helmets. For this measurement, the subject must be measured while in the Frankfort plane. When this measurement is compared to the actual top of the head while wearing a helmet, the measurements would be dramatically different. Using the Frankfort plane to determine the location of the vertex would provide inappropriate measurements for helmet design. Robinette and Whitestone (1992, Figure 1)) provide a drawing of the problems caused by orientation during traditional measurements.

The third limitation expressed by Robinette (In-press) was that traditional anthropometric data might not be sufficient for design or re-design of products. Traditional anthropometry is dependent on landmarks. These landmarks may become hidden when subjects wear the protective helmet or other product. Since the landmark is not viewable, the spacial relationship between the landmark and the product can not be measured. Without this information, the designers of the product cannot determine if the product fits in an appropriate

fashion. Three-dimensional scanning can overcome this problem when the subject is scanned before and after donning the product. The two scans can be aligned which permits viewing the points, including landmarks, underneath the product that would otherwise have been hidden.

The fourth limitation of conventional anthropometry mentioned by Robinette (In-press) is that there can be large differences in measurements taken by different observers. Large variations in conventional anthropometric measurements have been seen between different observers (inter-observer variation) and between the same observer (intra-observer variation). These variations could be due to a number of sources or a combination of sources including different identification of landmarks, problems in measuring tools, or improper measuring techniques. Surface scanning technology overcomes this limitation by capturing sufficient surface points to provide an electronic replication of the subject under observation. Therefore, the identification of landmarks and the use traditional anthropometric methods becomes unnecessary to provide an accurate model of the subject.

2.2 Modeling of 3-D Form

All methods of modeling the shape and size of an object are aimed at presenting information about the characteristic way in which the object occupies space. The method chosen is usually based on a particular problem or purpose. When using a laser scanner for recording an object's form, a tremendous quantity of surface data points are collected. The technique for converting these

data points into useful mathematical models is referred to as surface or object reconstruction and is receiving a great deal of attention in engineering and medical fields. Surface reconstruction has applications in reverse engineering for product design and manufacturing, and in medicine for construction of artificial anatomical objects. In this section, I have focused the literature review on relevant methods for characterizing form for helmet design, rather than on a complete compendium of mathematical techniques for form description. The section describes the two currently most common methods for mathematically modeling form. The two methods discussed are: volumetric implicit functions including superquadrics and constructive solid geometry, and splines with emphasis on non-uniform rational B-splines (NURBS). Other form characterization methods such as Fourier analysis and the development of an energy matrix using the principles of kriging (see Piccus et al., 1993) are discussed in the next section on form comparison techniques. Before discussing the two mathematical form characterization methods, a few background paragraphs on computer 3-D visualization are provided below. Most of this information was obtained from the Hewlett Packard primer on computer graphics found on the internet at

http://www.hp.com:80/wsg/products/grfx/GraphicsPrimer/, and Jason Patterson's website found at

http://www.reflections.com.au/~jason/Articles/HistoryOfComputers/index.html.

For more information concerning computer 3-D graphics, the reader is referred to these two sources.

2.2.1 Computer 3-D Visualization

Computer-based hardware and software for displaying, modeling, and analyzing three-dimensional objects have experienced an explosion in growth and development in the last ten years. In 1989, Silicon Graphics introduced one of the first 3-D graphics workstation, the IRIS 4D Superworkstation. This workstation, along with the Silicon Graphic's lower-priced machines, were capable of interactive 3-D modeling and went on to dominate the 3-D graphics market. The availability of these high-powered 3-D graphics workstations allowed for the development of all sorts of interesting uses for interactive 3-D graphics. Applications such as 3-D computer aided design (CAD) and computer aided manufacturing (CAM), 3-D animation for special effects, and the introduction of the concept of virtual reality (VR) were developed using these machines. Today (1998), Silicon Graphic's machines are still the most popular computers used in 3-D visualization and modeling. However, other companies such as Hewlett Packard, Sun, and Digital have also introduced lower costing and higher performing machines which should increase the competition in the 3-D visualization market.

The usual purpose of 3-D computer visualization and modeling is to provide information needed to perform calculations associated with the process of geometric design such as the calculation of shape related properties. In

addition, computer visualization and modeling greatly enhances the engineering design environment by providing a method for creating and testing computer-generated prototypes. The representation of 3-D objects by computers is accomplished in a number of ways. The most common methods are by using polygons (also called wireframes), or by using mathematical equations for surface and solid modeling of the object.

Polygonal modelers combine polygons to build the objects. The points on the surface of the object are called vertices (points where the sides of the polygons intersect) and are represented in the computer by their spatial coordinates. Other characteristics of the polygonal models, such as the color of each vertex and the direction perpendicular to the surface at each vertex, called the normal, also may be specified. Since polygons do not create smooth surfaces, detailed models require an extremely large number of polygons to create an image that looks natural. Nevertheless, in all cases polygonal modelers only provide "curves" that are always approximations, produced from sequences of straight-line segments. However, polygonal models offer the advantage of computational speed (unless an extremely large number of polygons are used).

Computer programs that represent 3-D objects by using mathematical equations do this mainly by using boundary representation (b-rep) to model the surface of the object or by volumetric implicit functions. In boundary representation, each face bounding an object is defined explicitly. Boundary

representation methods create curved surfaces of the object by piecing together several short curves in segments. Some common curve representation equations for b-rep methods are B-splines and NURBS (Non-Uniform Rational B-Splines) (discussed below). B-splines and NURBS are blended piecewise polynomial equations passing near a given set of control points. B-splines use equally spaced points along a path for knotting the polynomial equations together, while NURBS use irregularly spaced points giving them extraordinary flexibility and smoothness. B-spline and NURB representations allow the object's curves to be stored as parameters, which reduces the amount of information necessary to describe a complex shape, and increases performance. Spline-based modelers offer greater precision than polygonal-based programs and are better at producing complex shaped objects with smooth flowing lines. Engineers, designers, artists familiar with using curves in two dimensions often find spline-based 3-D modelers easy to learn, and spline-based modelers seem to be dominating the computer 3-D software marketplace.

In contrast to b-rep methods of using splines, 3-D object representation by implicit volumetric functions produces objects that, like a brick, concrete slab, or wooden beam, have substantive characteristics defined by their spatial volume, as well as by the shape of their surfaces. Implicit volumetric equations provide complete, unambiguous mathematical representations of 3-D objects and are discussed in the next section.

2.2.2 Solid Modeling using Implicit Functions

A number of researchers are currently investigating methods for mathematically modeling geometrical form using advanced systems based on the principles of solid geometry and the use of implicit functions. Implicit functions are mathematical expressions in which the function of concern is not directly expressed but must be arrived at by manipulation of the expression. Geometrical objects, such as a cone, cube, cylinder, and sphere, may be modeled by simple implicit functions called "primitives", while more complex figures may be modeled by superquadrics or other volumetric functions. In addition, models for form have been developed by combining primitive mathematical expressions using Boolean operations such as union, intersection, and difference. This method of modeling is called constructive solid geometry and is very appealing due to its intuitive formulation (remember using building blocks as a child), and is directly analogous to manufacturing processes where complex solids are created by "cutting and pasting" together primitive solids. Some current research and the advantages of implicit functions for modeling geometrical form are described below.

2.2.2.1 Superquadrics

Superquadrics are simple mathematical expressions for 3-dimensional objects that are useful for geometric modeling. Superquadric surfaces are derived from quadric surfaces, but a new flexibility is achieved by raising each

trigonometric term to an exponent. The exponents control the relative roundness and squareness in both the horizontal and vertical directions. Like quadrics, superquadrics have well defined normal and tangent vector equations. Normal vectors are used in intensity calculations during computer rendering, and both the normal and tangent vectors are used to calculate the curvature of the surface. By adjusting a relatively few number of parameters in the superquadric mathematical equations, a large variety of shapes may be obtained. Barr et al. (1981) derive formulas for the representations, normal vectors, tangent vectors, and inside-outside functions for several parametric forms of superquadrics.

Ferrie et al. (1993) describe the use of superquadrics in fitting geometric shapes to laser rangefinder data. The laser rangefinder retrieves an estimate of surface points of the objects. Superquadrics are then used to model the surface region. In order to fit a superquadric to a surface region, 11 parameters must be determined. Ferrie et al. (1993) employ a two step modeling approach which include determining the initial fit for the surface region and then attempting to minimize the difference between the model and the actual data. The first step includes determining the appropriate rotation and translation parameters for the relative position of the superquadric. The second step uses an error minimization algorithm to adjust the shape and extent parameters of the superquadric to minimize the difference between the model and the actual laser scanning surface data. Terzopoulos and Metaxas (1991) extend the basic parameterized form of the superquadric to include local control parameters that

they name deformable superquadrics. Their model combines the flexibility of splines with the generality of superquadrics so that it may be possible to obtain a closer fit to the actual data.

Shiang (1993) used superquadrics for modeling the human cranium for evaluating cranial asymmetry. The study comprised 20 Amerindian crania collected prior to 1950 from post-Pleistocene burial sites in California and Virginia. A 13-parameter superquadric equation was used to model the cranial surfaces of the 20 Amerindians. Since using superquadrics provides a model that is inherently symmetric, asymmetry was quantified by the radial deviations of the actual individual data points from the best-fit superquadric model. All skulls demonstrated only subtle surface variations from the superquadric model with root mean square radial fidelities of approximately 1.2 millimeters. This indicated that a superquadric model could provide a highly accurate method of mathematically describing the human skull. The study also revealed that the superquadric model could be used for observing anatomical variation in that all skulls displayed marked asymmetry in the inferior posterior region of the cranial vault. Shiang (1993) also investigated the use of simple quadric ellipsoids for modeling the cranial form, but found that this method can only be used for approximating the shape and size and did not provide the accuracy needed for describing anatomical variation such as squareness or taper. When comparing the simple quadric ellipsoid model to the superquadric model, the superquadrics reduce the root mean square radial fidelities by about 50 percent, which

indicates that superquadrics are a more appropriate approach for modeling the skull's shape and size.

2.2.2.2 Constructive Solid Geometry

Researchers at the University of Washington have published a number of papers (Lim et al., 1995,1996, and 1997) concerning the surface reconstruction of laser scanning data to useful mathematical representation of object form. Their approach generates an implicit function, $f(x,y,z) = 0$, to represent the reconstructed object (see Ganter and Storti, 1993). The implicit function is obtained by approximating the Boolean unions of geometrical primitives that best fit the surface data points. The complete representation of the object is unambiguous and is associated with the ability to construct a point membership classification that categorizes any point in 3-D space as either belonging to the interior, exterior, or surface of the object. The results (Lim et al., 1995), using this method for reconstructing sample objects, such as the torus, molar, femur and vertebrae, demonstrate that the reconstructed models are accurate to within a few percent of the actual object's volume.

2.2.2.3 Advantages of Implicit Representation

Advantages of using the implicit representation are that the reconstructed object can be subjected to any regular Boolean operations, and the ease of generating new shapes allows the user to incorporate the resulting solid into other models. Further, with the implicit representation, the object is continuous everywhere, and the test for point membership classification can be

performed with an evaluation of the function. Implicit representation is also attractive for applications in which variations of objects are analyzed because the differences between successive solid forms can be easily determined. Another advantage is that 2-D sections of a solid can be obtained from the algebraic function of the 3-D solid by simply setting one variable to a constant value.

2.2.3 Modeling using Splines

As described previously, today most computer models for constructing and manipulating three-dimensional objects represent the surface of the object as either polygons, usually triangles, or as a series of curved surfaces. Since polygons do not create smooth surfaces, detailed models using polygons require an extremely large number to create an image that looks natural. For models using curved surfaces, the two most common methods for mathematically describing the form are by using implicit functions, described above, or by using parametric polynomial expressions, the most common being non-uniform rational B-splines (NURBS). As described earlier, the major advantage of implicit solid modeling is that the surfaces can be concisely and unambiguously represented, while a disadvantage is that free-form surfaces can not be easily characterized. Using NURBS one can easily represent free-form curves, but simple geometric shapes such as cones or spheres are more difficult when using NURBS than with implicits, and require extra computer memory to accurately describe the surface. In addition, models using NURBS are inherently complex due to the number of surfaces required to accurately describe an object, and the ensuring

geometric continuity of patched surfaces is difficult to formulate and inevitably a very tedious process.

Non-uniform rational B-splines have, however, become the most common method of representing curves and surfaces in computer-aided design (CAD) and computer aided manufacturing (CAM) programs. The reason is that they have provided a common mathematical form for representing and designing both simple primitive shapes and more complex and free form curves and surfaces. They are also computationally stable, reasonably fast, and very accurate for representing actual data points in solid reconstruction efforts. In fact, NURBS could be used to fit every data point exactly, but this would lead to a huge database and, more than likely, a wiggly and uncharacteristic model. The better method is to fit the data by an approximate surface that goes near most and through some data points, but does not pass directly through each point. Hsieh et al. (1993) using a spline approximation surface documented a maximum error of 2.14 mm for a human femur when 520 data points were compared to the model.

2.3 Mathematical and Statistical Techniques for Comparing Forms

This section reviews the mathematical and statistical techniques for comparing the 3-D form of objects. The section is divided into two parts: methods which rely on anatomical landmarks and methods which do not rely on anatomical landmarks.

2.3.1 Methods based on Anatomical Landmarks

Richtsmeier et al. (1992) provides a summary of the current techniques of studying anthropological form using anatomical landmarks. For our purposes, the form of an object involves both its size and shape. The word "morphometrics" is used by the authors to describe the statistical study of biological shape and shape change. Bookstein (1982) has also defined morphometrics as the "empirical fusion of geometry and biology." Richtsmeier et al. (1992) discuss four current anthropological morphometric techniques, which include finite-element scaling analysis, thin-plate splines, Procrustes analysis, and Euclidean distance matrix analysis, and Piccus et al. (1993) provide some simple examples using these methods. All of these methods rely on the identification of anatomical landmarks, and are commonly used when traditional anthropometric data are available. A brief outline of these methods follows.

2.3.1.1 Finite-Element Scaling Analysis (FESA)

Finite-element scaling analysis is a form of finite-element analysis widely used in engineering. Finite-element analysis is commonly used in civil engineering structural projects to model material strains under loads such as for high rise buildings or bridges. Lewis et al. (1980) and then Cheverud and Lewis et al. (1983) first reported a description of finite-element analysis for an application toward morphometrics. In morphometric applications, finite-element scaling analysis is a method of comparing the forms of at least two subjects. In

the FESA, an initial reference form and a target form must be available, both with a set of similar anatomical landmarks, that is, there must be a one-to-one correspondence between landmarks on the two objects. The landmarks in each object are connected by line segments to make a series of finite elements that model each form. FESA then determines the amount of "morphometric strain" required to produce the target model from the reference model. The morphometric strain is defined as the ratio of the difference between the homologous linear distances in the two forms under study divided by the linear distance in the reference form (Cheverud et al., 1983, p.157; Lazonoff, 1990).

Finite-element scaling analysis has been identified as extremely useful for the comparison of the differences between pairs of forms (Richtsmeier et al., 1992, p. 290). It has been used in a number of different applications for comparing morphological differences including, among other things, craniofacial growth, craniofacial abnormality, and artificial cranial modifications (see Richtmeier et al., p. 289). However, for modeling the human head for helmet design, it is not a useful method and has many problems including lack of appropriate statistical procedures on the FESA results and the dependence on landmarks.

2.3.1.2 Procrustes Analysis

Procrustes analysis (PA) is a method of comparing two or more forms by superimposing one onto another according to a specified minimization criterion (Richtsmeier et al., 1992, p. 291). The method attempts to match the landmark

coordinates on two forms as closely as possible. The name of the method comes from Greek mythology. Procrustes is a mythical Greek giant who stretched or shortened captives to make them fit his beds. Many algorithms have been developed for Procrustes analysis including minimizing the sum of the squared distances (that is, least squares) (Goodal and Bose, 1987), and minimizing the distance between landmarks that do not match exactly (Siegel and Benson, 1982, p. 344). Procrustes analysis may be most useful in comparing anthropological forms when a specific hypothesis is being tested about a localized morphological difference (Richtsmeier et al., 1992, p. 293). It has been used to study the evolution of fossil ostracods, egg size in marine invertebrates, and cell growth (see Richtsmeier et al., 1992, p. 292). For human head modeling for helmet design and sizing, however, it has not shown to be useful because it explicitly limits itself to landmark data and makes no assumptions about the area between the landmarks.

2.3.1.3 Euclidean Distance Matrix Analysis

In Euclidean distance matrix analysis (EDMA), a matrix of distances between all landmarks for a specific object is developed (that is, a form matrix) (see Lele and Richtsmeier, 1991; and Lele, 1991). To compare objects, a form difference matrix (FDM) is developed by division of the corresponding distances in the two form matrices of the objects. If the two objects have the same shape, the values in the FDM would all be the same. If the two objects have the same shape and size, all values in the FDM would be equal to one. When comparing

two groups of objects, an average form must be developed for each group by using a Procrustes algorithm. Euclidean distance matrix analysis has been used to analyze growth patterns within primate species and among patients with craniofacial abnormalities (see Richtmeier et al., 1992, p. 295). It has not however been used in analyzing human heads for helmet design, because, like Procrustes analysis and finite element scaling analysis, it is a one-to-one comparison method and depends on anatomical landmarks.

2.3.1.4 Thin Plate Splines

Thin plate splines have been used as a mathematical method for evaluating the deformation of landmarks on one object to the corresponding landmarks on another object. Splines are parameterized polynomial equations which, in anthropological applications, pass through the anatomical landmarks of the object. Although there are infinite number of equations that would pass through the landmarks, the spline is the expression that requires the minimum curvature or bending energy.

Bookstein (1989) was one of the first to consider the application of thin plate splines for comparing anthropological objects. His idea is to compare two objects by evaluating the function required to deform one object's landmarks to fit the corresponding landmarks on the other object. The major disadvantages of this method are that the procedure is landmark dependent and comparisons can only be made between two objects. Since comparisons can only be made

between two objects, there is a need to define a standard object form from which all other objects can be compared.

2.3.2 Methods not Based on Anatomical Landmarks

Using methods based on anatomical landmarks for comparing anthropological forms has many limitations. The first and most important limitation is that curvature and other features of the surfaces between the landmarks are lost in the analysis. Secondly, landmarking requires a great deal of physical effort, and on the human head there are not a great many clearly identifiable points which can be located without palpation of the skin. In the past, and probably still today, landmarking and using traditional anthropometric measurements (discussed previously) were the most widely used methods for characterizing and comparing the human form. However, today the measuring tools also include the digitizing technologies (described previously) which can gather hundreds of thousands of surface data points of the object in a matter of seconds. For head anthropometry, the laser scanner can collect greater than 200,000 points on the surface in less than 20 seconds. To overcome the limitations of using landmarks and the shortcomings of traditional anthropometry, and to take advantage of the new digitizing technology, methods for characterizing and comparing biological shape and size that do not rely on landmarks are desirable. These methods are reviewed next.

2.3.2.1 Statistical Analysis of Parameters of Volumetric Implicit Functions

Applications of the use of volumetric implicit functions, such as those developed from constructive solid geometry, for characterizing the human head are not yet numerous. Therefore, statistical analysis of the parameters determined for the implicit functions for comparing the size and shapes of the objects is scarce. The only source which could be found was work done by Shiang (1993) (discussed previously, see section on superquadrics). In his study, only descriptive statistics of the 13 parameters of the superquadric models were provided for 20 skulls of the Amerindians. No multivariate analyses or clustering routines were conducted because they were not needed for evaluation of the asymmetry of the skulls which was the focus of the study. No other studies investigating the use of implicit functions for modeling human heads were identified.

A simple variable for clustering the human head for sizing of helmets could be the estimated volume of the head. Volume (or mass) is one natural measurement, which probably has some relationship to the grouping of individuals for helmet sizing. Univariate statistical analysis could be accomplished on the head volume data to develop the groups for each size. However, the shape of each the heads would be disregarded in the analysis, which would, more than likely, produce poor results. No studies using head volumes for grouping for the sizing and design of headgear were identified.

2.3.2.2 Fourier Methods

The Fourier transform is a standard mathematical technique that has been used in many engineering, medical, other scientific applications such as in the study of the physics of vibrations and optics. It has also been used as a technique for modeling geometrical form, mostly in two dimensions, but more recently research has been conducted using Fourier methods for 3-D objects (see Szekely et al., 1996; Park and Lee, 1987). In shape and size analysis, the Fourier transform's aim is to convert a description of a complicated geometrical form to a new mathematical expression in which the form is regarded as built up by a series of sine and cosine functions. This transformation to the Fourier equation results in a much simpler expression that consists of two basic components: the Fourier coefficients (pairs of sine and cosine coefficients) and the frequencies associated with them. Each pair of sine and cosine coefficients represents the relative contribution at the associated frequency in describing the form of the object.

Only one article was found concerning the use of Fourier methods for analysis of form for the design of protective equipment. Ratnaparkhi et al. (1992) used the Fourier transform to model 2-D horizontal cross-sections of the human head. In this study, the Fourier transform was used to model the surface contour of the cross-section, and then the Fourier coefficients were used in a cluster algorithm for the determination of groups for sizing. One difficulty noted by the authors was in the selection of the Fourier coefficients that are most useful for

discriminating the shapes. Another difficulty noted was the fact that very different entities, such as the ear and the nose, may contribute to the same Fourier coefficient on one individual and not to another. However, the major advantage of this method is that the Fourier coefficients are insensitive to the starting point of the analysis of the surface contour, which makes it an applicable method when not using anatomical landmarks. Also, the Fourier transform produces coefficients which are orthogonal (that is, uncorrelated) which allows for comparatively straightforward statistical analysis.

2.3.2.3 Energy Matrices

Piccus et al. (1993) and Robinson (1994) describe a method for characterizing form based on the development of an energy matrix for that form. This characterization method does not rely on the use of landmarks and can be applied to unequal size data sets (that is, the number of points identified on the object's surface can vary from object to object). The energy matrix developed from this method can be thought of as the bending energy required to transform a flat plat to the shape and size of the object. The energy matrix is specific for each object in that it uniquely characterizes the set of points used to describe the object's form. Since the energy matrix is unique for each form, it is a source for variable selection and comparison using statistical methods. The major disadvantage of the method is the existence of a large data set, combined with the uncertainty of how to identify the critical features of the energy matrix for

comparison (this is also a disadvantage in the Fourier methods described above).

Piccus et al. (1993) and Robinson (1994) describe the use of this energy matrix form characterization method for the sizing and design of flight oxygen masks. The energy matrices for the facial areas of 99 individuals were developed. These matrices were then subjected to principal component analysis to generate an orthogonal, uncorrelated sample for each object. The new sample sets from this transformation were then used in a fuzzy clustering algorithm to develop the groups of individuals for the sizes of the oxygen masks. Those subjects whose faces had the highest degree of membership in each of the clusters were chosen as the prototypes for each of the mask sizes. The masks were then manufactured and distributed to a sample of subjects for testing by a qualitative survey. The results of the qualitative evaluation were mixed and the authors noted the degree of membership was not in itself an accurate measure of fit (Robinson, 1994, p. 54).

2.4 Historical Development of Flight Helmet Designs and Sizing Systems

This section provides an historical review of the design strategies and methods of sizing aviation headgear. The section has three parts. First, a discussion of the early headgear designs and sizes is provided, followed by a description of some of the first studies of methods using anatomical landmarks and traditional anthropometry. The last part discusses the more recent design and sizing methodologies for flight helmets.

2.4.1 Early Designs and Methods.

The design of headgear for aviation began in the early days of flying. Soft leather helmets were the first designs and were the headgear of choice for World War I and II aviators. These helmets protected the wearer from wind, rain, and cold, as well as engine noise, but provided little crash protection. These soft helmets usually came in only one size and had a drawstring or cloth strap for adjustment around the face and neck for a closer fit. Hard helmets, such as football helmets procured by the Army before WWI, and the Roold protective aviator helmet, were available, but the pilots learned quickly that these helmets did not allow the freedom of movement that was so important in combat flying. Three good sources of information and photographs of the early flying headgear are Vintage Flying Helmets by Mick J. Prodger, Combat Flying Clothing by C.G. Sweeting, and Flying Clothing by Louis Greer and Anthony Harold.

The testing of the designs of crash protective headgear, that is, hard helmets, for military aviators began in 1943 at the Army Air Force (AAF) Personal Equipment Laboratory (Sweeting, 1984, p. 79). William L. Moore, an engineer at the laboratory, personally tested most of the early designs by donning the sample helmet and hitting himself on the head with a mallet and banging his head on a post (Sweeting, 1984, p. 79). No hard helmet was standardized during the war, but by 1945, AAF pilots were making improvised hard helmets for use with the new jet aircraft. These hard helmets were needed

because of the buffeting the pilots encountered while flying at high speeds through turbulent air (Sweeting, 1994, p. 79).

2.4.2 Methods Using Traditional Anthropometry

One of the first documents that addresses a method of developing sizes for flight helmets based on traditional anthropometric measurements was provided by Zeigen, et al., in 1960. This report presents a method for sizing and design of rigid and semi-rigid helmets and helmet liners based on a single key dimension, head circumference. Anthropometric data obtained largely from a 1950 survey of Air Force flying personnel were analyzed and used in the development of the sizing system. Head circumference was used to divide the population into groups. The average values of the traditional anthropometric measurements, such as head length and breadth, in each of the groups were then used to produce artificial headforms. Sculpturing techniques were used to make the artificial headforms using plaster of paris as the medium.

Ziegen et al. (1960) also report two earlier sizing systems for flight helmets and helmet liners which were developed in the 1940s and 1950s. The first was a method that used head circumference, as in their sizing system, which was used for design and sizing of soft leather helmets. This method was described as the "first attempt by the Army Air Force to objectively develop standard headgear." Anthropometric data from the Army Air Force cadets were used to establish a four-size system in which four artificial headforms were sculptured in 1944. These headforms, used as quality control gauges, were

fabricated to the mean values of 18 dimensions including a number of facial ones. The design criteria used in these headforms was also used in the development of the sizing systems for the M-1 Army helmet (Zeigen et al., 1960, p. 5).

The other early sizing system described by Zeigen et al. (1960) was the 1954 Head Length-Head Breadth System. This system was the first for rigid crash helmets for protection from buffeting, and was needed because of the inadequacies of the 1944 Headform Series mentioned above for the soft leather helmets (Zeigen et al., 1960, p. 6). The 1944 Headform Series was described as inadequate because: "(1) it was based on measurements made on an Air Force population known to be significantly different than that measured in 1950; and (2) the problem related to rigid, non-close fitting shells, requiring joint consideration of the head and face, a matter not of prime concern in the 1944 Series." The 1954 Head Length-Head Breadth System used traditional anthropometric data from a 1950 survey of Air Force flyers. This system provided 16 artificial headforms for the development of helmet liners. These artificial headforms were based on six head length intervals each combined with three cephalic index groups. The cephalic index is a percentage relationship between the head breadth and head length. The headforms were sculptured using 41 traditional anthropometric measurements. The major drawbacks of the 1954 System were described as its complexity and limited applications to designing other types of

helmets (Zeigen et al., 1960, p. 7). It was also described as providing helmets which were bulky and uncomfortable (Zeigen et al., 1960, p. 7).

2.4.3 More Recent Methods of Design and Sizing

More recent developments of a helmet design and sizing methodology have included works by Sippo and Belyavin (1991), Robinette and Whitestone (1992), Bradtmiller and Beecher (1993), and Whitestone and Robinette (1997).

Sippo and Belyavin (1991) describe a method of sizing using three key dimensions: head length, head breadth, and pupil-vertex height. The paper states that pupil-vertex height is a required key dimension for a helmet sizing system because of the new helmet-mounted display technologies such as night vision goggles. The authors use multivariate statistical analysis on anthropometric data from a 1970/1971 survey of 2,000 Royal Air Force Aircrew. The authors show that a nine size system, with one size centered around the mode of the trivariant data (that is, head length, head breadth, and pupil-vertex height), results in theoretically 93.5% of the population having at least an adequately fitting helmet. Quality of fit for the sizes developed from the author's sizing methodology was assessed by determining whether the head's dimensions were within acceptable tolerance levels for the three key dimensions. Sippo and Belyavin assumed "that the topological peculiarities of individual human heads would be relatively unimportant to basic design plans and that such differences would be eliminated by some type of form-fit suspension system." Due to this assumption, the paper does not discuss how the shape and

design, or the other dimensions of the helmet sizes will be determined. Their sizing scheme only specifies certain ranges for the three key dimensions of head length, head breadth, and pupil-vertex height.

Robinette and Whitestone (1992) provide, for the first time, a discussion of the use of three-dimensional anthropometry using surface laser scanning in the design of helmet systems (also see Whitestone, 1993; and Robinette and Whitestone, 1994). Their report describes two approaches for characterizing the human head in the design process. Their first approach is for the design of new, prototype helmets, and their second approach is for the modification of existing helmets or in the redesign of prototype helmets. The first approach has three basic steps: the digitization of the head and face surfaces of a sample of subjects from the population of interest, the statistical selection of a small number of representative cases from these, and the creation of three-dimensional forms of these candidates (Robinette and Whitestone, 1992, p. 6). The authors select head length and head breadth to divide the population into groups. Then the authors arbitrarily select target head length and head breadth points within these groups for selection of heads to represent the group. The selected head must meet a tolerance level of plus or minus four millimeters of the target points. The three-dimensional data from these selected heads were used for preparation of artificial headforms by construction with a numerically controlled milling machine and for two-dimensional scale drawings of selected perpendicular slices.

The second approach for design of helmets presented by Robinette and Whitestone (1992) requires the use of an existing helmet system and investigates the locations of key head features, such as the pupil, with respect to that helmet system. The approach includes scanning an individual with the helmet on and with the helmet off. The helmeted scans, when linked with scans without the helmets, provide the designer with the ability to see where the head and its key features are with respect to that specific helmet system. Large gaps between the helmet and head may indicate that the helmet is too large for the wearer, while tight fitting helmets would show very small gaps. Also key features, like the locations of the pupil and the ears, can be visualized with respect to the helmet.

Bradtmiller and Beecher (1993) provide a preliminary report describing an approach to creating three-dimensional headforms for helmet sizing and design. In their approach, vectors from a center point in the head to the points on the surface were calculated from a data set of 475 males and 462 females in a nationwide sample. The authors state that the vectors form variables that can be manipulated in traditional ways and that they are pursuing two analytical approaches with the radii. One of their approaches is cluster analysis, and the other is a traditional sizing approach that will account for variability within a size and target the standard head forms to the larger individuals within the size. The authors do not provide the results of their statistical analysis using clustering or the traditional approach.

Whitestone and Robinette (1997) published the most recent description of methods for developing the designs and a sizing system for helmets. Their chapter in a book on head mounted displays is entitled "Fitting to Maximize Performance of HMD Systems." The authors address a variety of issues including both design and sizing methods that work and that do not work. They argue that the use of univariate statistics and percentiles, the Frankfort plane, present definitions of line of sight, and the use of anthropometry for grouping individuals into sizes all are methods which do not work for designing helmets. Methods that do work the authors argue are three-dimensional scanning and fit testing. The authors discuss the scanning of helmeted and unhelmeted individuals and a method of registering these scans to provide an "x-ray" view into the helmet so that fit geometry can be visualized. The authors also provide commentary on the future directions and research needs for helmet design and sizing including defining the line of sight, the generic head alignment, and a biofidelic computer-aided design head.

CHAPTER 3

3 Basis for Design and Sizing Methodology

This chapter discusses the basis for the methodology purposed for the design and sizing of close-fitting protective helmets such as those worn by fighter aircraft pilots. The chapter describes the methodology including the development of the vectors for design and the statistical analysis for clustering the heads. It includes the reasoning behind the use of vectors and compares this method to other techniques currently used for design and sizing of protective helmets. The chapter includes a short introduction with a review to summarize the need for the methodology, and then a detailed explanation of the methodology and its justification.

3.1 Introduction

As discussed in Chapters 1 and 2, laser scanning technology for three-dimensional surface digitizing has provided designers of protective equipment and clothing another tool of anthropometric measurement for optimizing fit and improving performance. This technology can provide in a matter of seconds hundreds of thousands of accurate, three-dimensional surface coordinates of anatomical objects. These 3-D surface points can then be visualized, measured,

and manipulated for the design of many devices which require close fitting contact with the human body. One such device is the fighter aircraft pilot helmet. Fighter aircraft pilot helmets, like most high impact protective headgear, must be very close fitting and stable on the head to prevent the helmet from dislodging during impact making the head vulnerable to injury. It is also is desirable to reduce the mass and moments of inertia of these helmets to prevent neck strain and reduce the risk of neck injury during an accident. Recently introduced helmeted mounted displays, such as night-vision goggles or optical targeting devices, only add to the weight and moments of inertia increasing the risk of instability and failure of the helmet system. Therefore, the next generation of fighter pilot helmets will need to be extremely light, close fitting, and highly stable to provide optimum performance during military operations and furnish the maximum protection of the head during accidents.

Until recently, close-fitting protective helmets have been designed using traditional anthropometry and univariate and bivariate statistics of linear measurements of the head. Traditional anthropometry uses soft measuring tapes, calipers, and other hand-held devices to take one-dimensional measurements of the human body. These traditional measurements are performed on overall dimensions of the head or between anatomical landmarks, which are homologous body surface points with biological correspondence from subject to subject such as the tip of the nose (named pronasale). Current designs of fighter aircraft pilot helmets were developed using head length and

head breadth, and then setting the dimensions of the helmet to meet specified percentiles of the target population based on these measurements. By using only a few measurements of the head to design the helmet, no information is employed in the design methodology on the shape of the human head. Therefore, the remaining dimensions of the helmet are provided from an artistic reconstruction of the head's shape and size. To accommodate a majority of the helmet wearers, this methodology causes the non-quantified dimensions to be larger than necessary resulting in needless space between the helmet and the head. This empty space results in an increase in the mass and moment of inertia of the helmet subsequently causing more instability and expanding the risk of failure of the helmet system.

Recent developments in helmet design and sizing methodologies were discussed in the literature review of this dissertation, and included works by Sippo and Belyavin (1991), Robinette and Whitestone (1992), and Bradtmiller and Beecher (1993). To review, Sippo and Belyavin (1991) describe a method of sizing using three key dimensions: head length, head breadth, and pupil-vertex height. The authors show that a nine size system, with one size centered around the mode of the trivariant data, results in theoretically 93.5% of the population having at least an adequately fitting helmet. Quality of fit was assessed by determining whether the head's dimensions were within acceptable tolerance levels for the three key dimensions. Sippo and Belyavin assumed "that the topological peculiarities of individual human heads would be relatively

unimportant to basic design plans and that such differences would be eliminated by some type of form-fit suspension system." Due to this assumption, the paper does not discuss how the shape and design, or the other dimensions of the helmet sizes will be determined. Their sizing scheme only specifies certain ranges for the three key dimensions of head length, head breadth, and pupil-vertex height.

Robinette and Whitestone (1992) provide, for the first time, a discussion of the use of three-dimensional anthropometry in the design of helmet systems. The authors describe two approaches for characterizing the human head in the helmet design process. Their first approach is for the design of new, prototype helmets, and the second approach is for the modification of existing helmets or in the redesign of prototype helmets. The second approach includes transposing an unhelmeted 3-D scan of the head with the helmeted 3-D scan to visualize and determine the spacial area between the helmet and the head {see literature review above or Robinette and Whitestone (1992) for further details on this technique}. The methodology for sizing and design discussed by the authors for new helmets has three basic steps: the digitization of the head's surface for a sample of subjects from the population of interest, the statistical selection of a small number of representative heads from this sample, and the creation of three-dimensional forms and two-dimensional cross-sectional scale drawings of these representative heads for helmet design and manufacturing (Robinette and Whitestone, 1992, p. 6). The authors select head length and head breadth to

divide the population into groups. Then the authors arbitrarily select target head length and head breadth points for selection of the heads to represent the sizes. The three-dimensional data from these selected (i.e., representative) heads were then used in the preparation of artificial headforms and two-dimensional scale drawings. This methodology for helmet design only uses two variables, which are head length and head breadth, for defining the groups for the helmet sizes. The selection of the target values and the representative head for determining the other dimensions of the helmet are arbitrarily chosen. This method will result in poor fitting helmets for a majority of individuals because the shape and size of the representative head are different from the shape and size of the other member's heads in the group. Further discussion of this method is completed in Chapter 5.

Bradtmiller and Beecher (1993) describe for the first time an approach that uses the 3-D surface point data from laser scanning technology for defining the sizes of headforms for helmet design. In their approach, vectors from a center point in the head to the points on the surface were calculated. The authors state that the vectors form variables, which can be manipulated in traditional ways, and that they are pursuing two analytical approaches with the radii. One of their approaches is cluster analysis, and the other is a traditional sizing approach that will account for variability within a size and target the standard head forms to the larger individuals within the size. The authors do not

provide the results of their statistical analysis using clustering or the traditional approach.

Huggins (1997) provides the most recent discussion of an approach for design and sizing of helmets using 3-D laser scanning data. His approach is to develop an aggregation of the head scanning data and formulate helmet sizes based on the percentiles of the aggregation. The head scans are aligned by using two or three anatomical landmarks. If only two landmarks are used, then two axes are aligned and the third axis is in an "as scanned" position. The author warns that the use of percentile shells should be approached with caution. He states that:

> "If the whole population in the data base is used to produce a series of percentile shells, which can then be used to determine the boundaries of certain sizes of helmets, then it is suggested that there would be some subject that will not fit into particular size roll and would have to be accommodated in a larger size. It is generally recognized that no such person as Mr(s) 50th percentile exists and that everyone is made up of a number of these percentile shells."

Huggins suggests that a bivariate plot of the key head dimensions (i.e., head breadth and head length) be used to divide the population into sizes. The resulting set of subgroups of the population would then be used for the design of the helmet.

3.2 Basis for the Use of Vectors and the Midpoint to Glabella Alignment System.

Many types of protective equipment and clothing can be designed using only a few key dimensions of the body. The reason is that the other dimensions

necessary for design of the article are well correlated with the key dimensions (that is, they can be predicted from the key dimensions with enough accuracy to ensure fit). For helmet design, the key dimensions have historically been head length and head breadth. However, the other traditional anthropometric measurements of the head are not well correlated with head length and head breadth. Table 3-1 provides the correlation coefficient matrix for traditional measurements of the head. The table shows that head length correlates well with head circumference, but poorly with the other dimensions. Head breadth correlates moderately with circumference

Table 3-1: Correlation Coefficients (Pearson's) for Traditional Anthropometric Measurements

	Circumference	B-C Arc	Breadth	Length
B-C Arc	0.565			
Breadth	0.589	0.591		
Length	0.857	0.353	0.278	
Height	0.386	0.703	0.280	0.306

and bitragion-coronal arc, but poorly with the other traditional anthropometric measurements. Appendix A provides summary statistics of traditional anthropometric measurements for the subjects from our sample from the 1990 U.S. Air Force Anthropometric Survey of the flying population. To graphically show the degree of correlation between the variables, Figures 3-1 through 3-4 are scatter plots with regression lines for various traditional measurements.

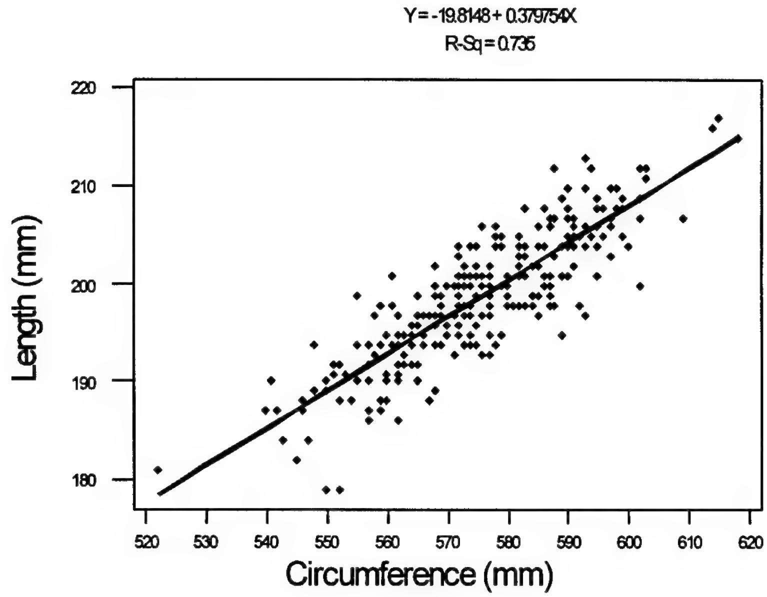


Figure 3-1. Scatter Plot with Regression Line of Head Length and Head Circumference.

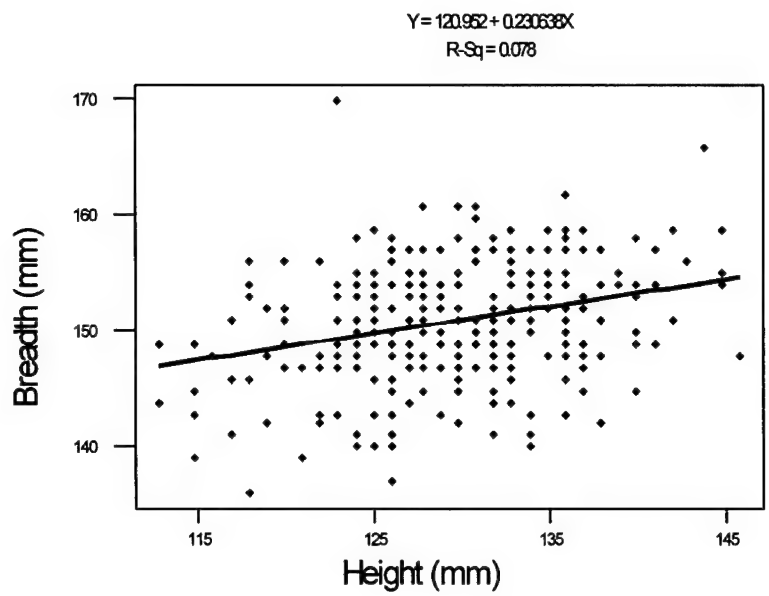


Figure 3-2. Scatter Plot with Regression Line for Head Breadth and Head Height

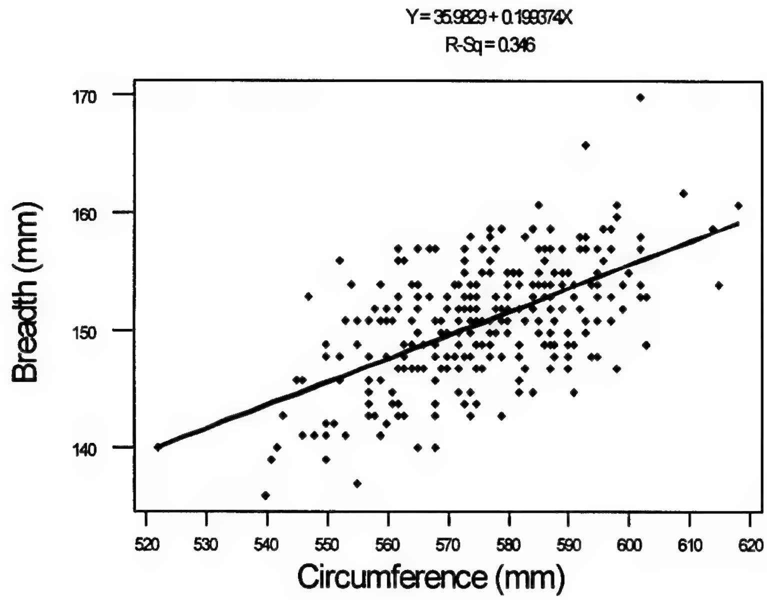


Figure 3-3. Scatter Plot with Regression Line for Head Breadth and Head Circumference

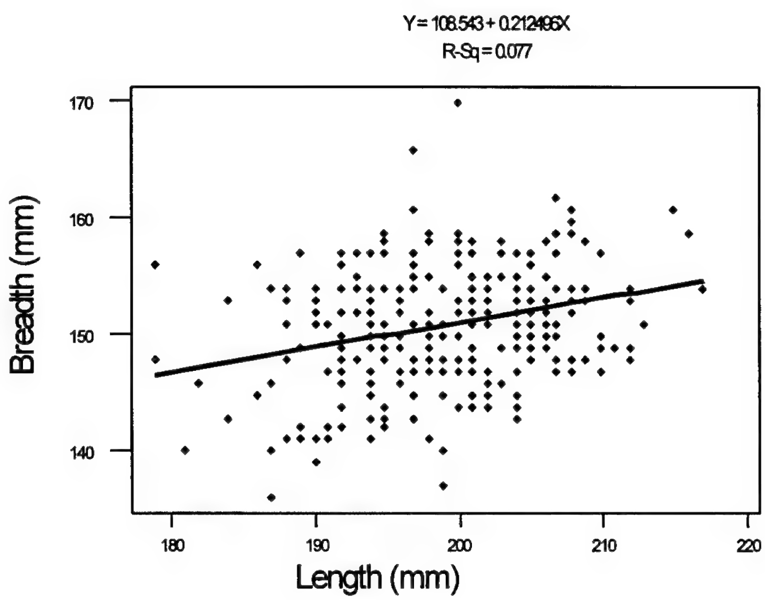


Figure 3-4. Scatter Plot with Regression Line for Head Breadth and Head Length

Presently-used flight helmets were designed using head length and head breadth for the key dimensions, while the other dimensions needed for design were formed by an artistic reconstruction of the head's surface. Other traditional anthropometric measurements of the head were rarely used in the design because they were not well correlated, and also because they could not be configured in an alignment system which provided results consistent with the design of helmets. For example, Whitestone and Robinette (1997) showed that the traditional alignment system of using the Frankfort plane, provided a very different measurement for the top of the head when compared to the actual top of the head while in the helmet. Because of these reasons, an alignment system is needed so that it is consistent with the design data needed for helmets. The use of vectors and the midpoint to glabella alignment system should provide this consistency.

A vector is defined as a quantity having both a magnitude and direction. For our purposes of comparing the shapes and sizes of heads, a vector can be thought of as a directed line segment with points being the midpoint of the head (starting point) and the surface point of the head (ending point). Since vectors do not have a reference to location, our directed line segments can be compared from one head to the next and statistical analysis can be performed on the vectors' magnitudes. To compare magnitudes between heads, the direction from one vector for one head must be homologous to the vector on the other head.

That is, one should only compare the magnitudes of the vectors from one landmark to another landmark, or from one anatomically identical location (for example, a midpoint) to another anatomically identical location without reference to landmarks. To statistically summarize the heads for helmet design, the heads must be oriented into an alignment system so that they all conform to a coincident axis system or the vectors direction angles must be related in some fashion.

Scanned heads from the Cyberware equipment puts the image in a different alignment for each head even though measures are taken so that the heads are close to a similar alignment. To compare one head to the next or to statistically summarize a group of heads, all the heads must be in a similar alignment system. This can be done by aligning two or three anatomical landmarks as described by Huggins (1997) or by aligning the head within a helmet system as described by Robinette and Whitestone (1992). Another approach is to define a vector for each head in which all other vectors for that head are compared. This is similar to alignment by two anatomical landmarks. The approach used in this research study is to define a vector for each head and then compare the direction of that vector to all other vectors in the head. This vector chosen as the reference is defined as beginning from the midpoint and ending at the glabella.

The midpoint to glabella was chosen as the reference vector to which all other vectors in the head are compared because of helmet design

considerations. For all wearers of helmets, the optimum location for protection of the head is to have the opening of the helmet for viewing just above the eyebrows. Helmets that cover below the eyebrows may block vision, while helmets that cover only a small portion of the forehead would cause an increased risk of injury to the uncovered area during an accident. The glabella, which is positioned just above and between the eyebrows, was the appropriate landmark for the endpoint of the reference vector for helmet design. In addition, it is desirable to have the eyes located centrally within the opening of the helmet. The glabella provided this point since the eyes are evenly positioned from the glabella.

A midpoint within the scanned 3-D data was chosen as the starting point for the reference vector because of geometrical considerations. A point centrally located within head provides a more accurate description of the head from the vectors, because it provides an even distribution of direction angles for the vectors. A midpoint has surface points surrounding it in all directions and the number of points in any one general direction should be approximately equal to any other general direction. If a point other than the midpoint were selected as a starting point, the number of surface points from the scanned head in one general direction would be unequal to the points in the other directions.

3.3 Defining A Midpoint in the Head

The scanned image of the head includes areas not covered by the helmet such as the face and portions of the neck. To define a midpoint based on

these areas as well as the areas covered by the helmet would be inappropriate from a helmet design standpoint. A more appropriate midpoint should be based solely on areas of the head that are covered by the helmet. For child bicycle helmet design, Bradtmiller and Beecher (1994) defined a midpoint for vector development on a plane which lies just above the ears and the glabella. On this plane, the midpoint would be mid-way between the anterior and posterior surfaces of the plane and mid-way between the lateral edges of the plane. The authors did not describe how exactly the midpoint plane would be defined (that is, where above the ears and the glabella would the points be for defining the plane). A problem with defining a plane based on the ears is that some individual's ears are larger than others are, and their locations on the head vary from individual to individual. For example, if a person has very large ears that are located high on the skull when compared to the glabella, then the midpoint plane would be angled upward from the glabella, and may cause the midpoint to be located higher in the head than it should be.

This dissertation research has investigated three methods for defining the midpoint of the head for helmet design. The first method investigated was based on the anatomical landmark the glabella (in the front of the head between the brow ridges, directly above the nose) and the farthest distance from the glabella in the horizontal plane. However, this method for determining the midpoint did not work well because of errant points in the scan data that were not part of the head's surface but were obtained from the scanning process. To

Figure 3-6. Placing Blue Dots on Anatomical Landmarks prior to 3-D Digitization of the Head

The computer program INTEGRATE 1.25, developed by the Computerized Anthropometric Research and Design (CARD) Laboratories, and the computer program SCULPT, developed by Dr. David Alciatore, were used to locate the holes in the point data and determine the Cartesian coordinates of the landmarks.

After determining the Cartesian coordinate data of the glabella and nuchales, the equation of the line that connects these two points is determined and the vertical plane passing through this line is defined. This plane is

considered the mid-sagittal plane. Points in this mid-sagittal plane that lie above the nuchales and are near the horizontal plane in which the glabella is located are determined. Among the points identified, the maximum point from the glabella is chosen for determining the midpoint of the head. The point that lies midway between this maximum point and the glabella is identified as a midpoint of the head. Figure 3-7 is a drawing depicting the location of this midpoint and is referred to as Midpoint Method 1 throughout the rest of this dissertation.

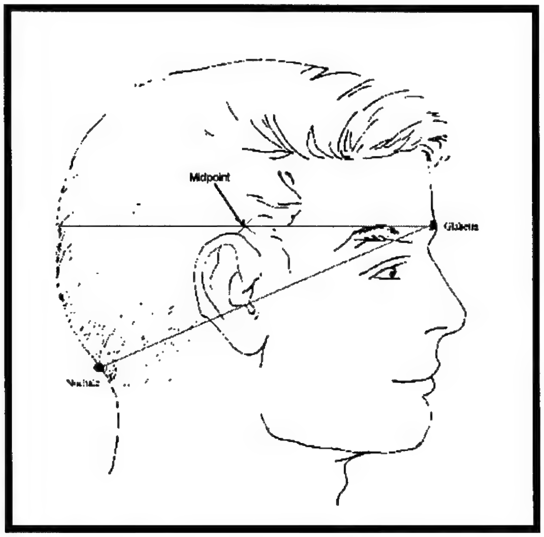


Figure 3-7. Midpoint based on Glabella and Nuchales, referred to as Midpoint Method 1.

The third method for determining the midpoint is more complicated than the first two methods, and was formulated because of problems encountered with the second method. For the second method, both the glabella and nuchales landmarks should line on the midline; however, in the real world this turns out to be not the case. When visualizing the data, the nuchales was located slightly left or right of the midline. This was probably due to errors when physically

landmarking the individuals prior to the scanning process. Therefore, a more complicated, third method was investigated for determining the head's midpoint. The procedure consists of first determining the equation of the line and the midpoint of this line between landmarks near the left and right ears. These landmarks are called the left and right tragions. Next, the vertical plane intersecting the glabella and nuchales is determined; and a second line in the horizontal plane of the glabella and in this vertical plane is defined. The midpoint of this line is then used to define a third line. This third line is defined as passing through this midpoint and parallel to the line defined by the tragions. A fourth line is then defined as a line perpendicular to the line connecting the tragions (the first line) and passing through the tragion midpoint. The intersection of the third and fourth lines is defined as the midpoint of the head. Figure 3-8 depicts this midpoint in the head, which will now be referred to as Midpoint Method 2. Vectors were then calculated from this midpoint to the surface points.

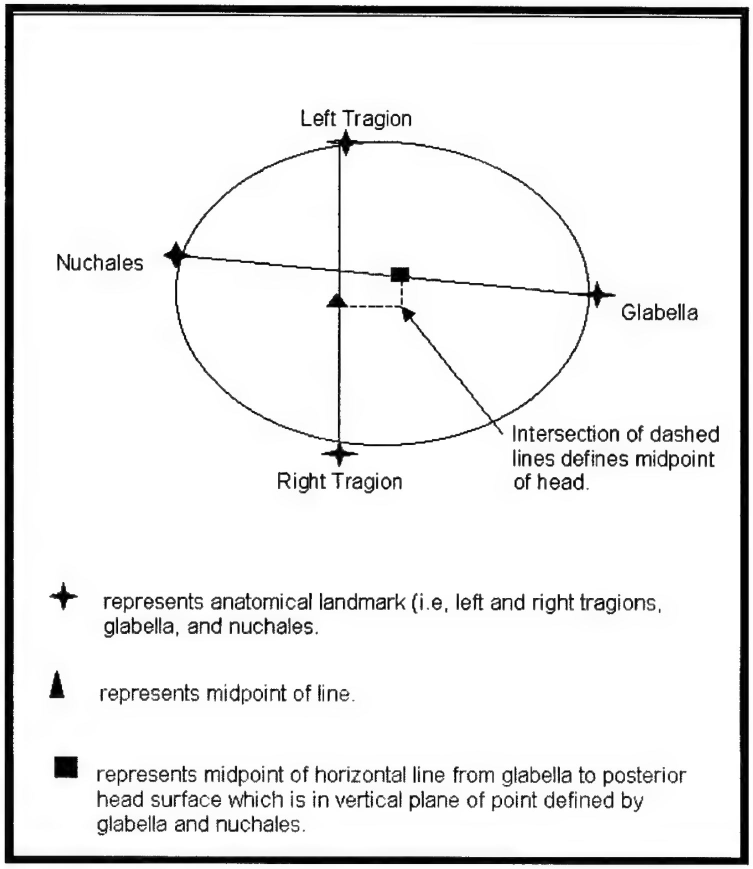


Figure 3-8. Midpoint Method 2. Illustration of a horizontal cross section of the head projected from the glabella.

The first method of defining the midpoint was abandoned due to the problems of the obtaining errant points not located on the head's surface from the scanning process. The second and third methods described above were used in calculating the vectors and a statistical comparison of these methods is presented in Chapter 5, Results and Discussion.

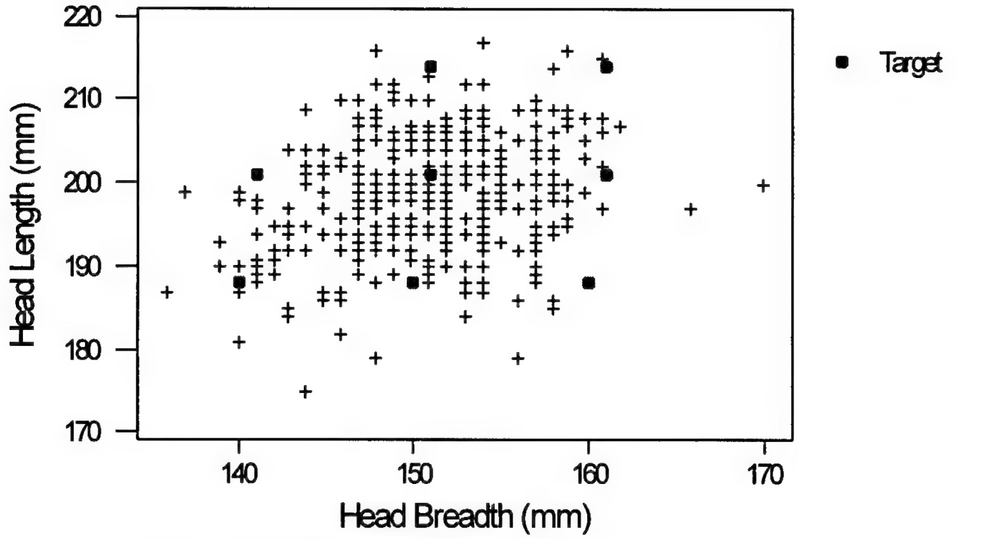
3.4 Development of a Sizing System

The goal of any sizing system for protective equipment and clothing is to group individuals by similar shape and size. This goal is based on the

assumption that individuals of a similar shape and size can wear the same size of a particular piece of equipment or clothing. The number of sizes of the article should be based on the degree of fit required for proper performance of the item. The number of sizes should increase for equipment that requires a close fit for proper performance, while the number of sizes for loose fitting articles, such as some types of clothing, should be less. However, it is usually not until after the sizes have been established and the product manufactured that the problems with fit are uncovered, unless extensive fit testing of the prototype equipment is completed prior to manufacturing. For relatively expensive items, such as helmets or respiratory equipment, it is desirable to have as few sizes as possible to reduce the costs of manufacturing, storage, and distribution. However, this presents a problem from a fit standpoint -- the fewer the number of sizes, the less chance of fitting a large majority of the population. Therefore, contrasting goals are present: reduce the number of sizes available while fit a large portion of the population.

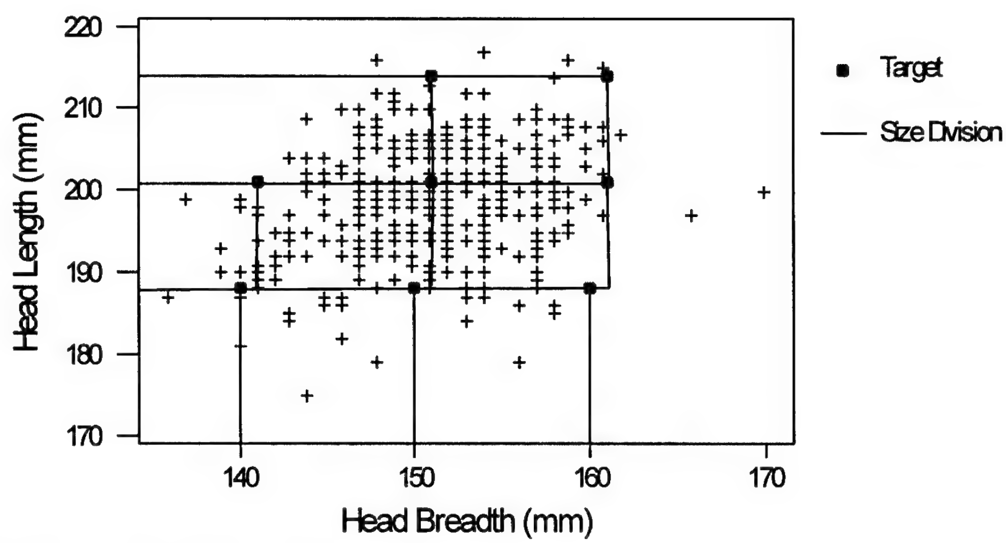
Present day methods of sizing of helmets is based on two linear dimensions of the head, which are head length and head breadth. Robinette and Whitestone (1992) discuss the most recent anthropometric survey of the Air Force flight helmet wearing population (1990). They provide a bivariate plot of head length and head breadth for a sample of male flight helmet wearers and then choose target points on this plot for design of helmets. Figure 3-9 is a reconstruction of this plot displaying the target points. The target points were

chosen to spread out across the distribution so that a large majority of helmet wearers would be accommodated. Individuals from this population who were within plus or minus 4 millimeters from the target points were then selected to represent the size. The head scans of these individuals were then used to develop two-dimensional scale drawings for helmet designers. Eight target points were selected, therefore eight sizes are proposed. The target points are in essence cut-off values for head length and head breadth, which is shown in Figure 3-10. Individuals who have a larger head length or head breadth measurement than the target head would need to be accommodated by a larger size if one exists. For individuals who have a head length greater than 21.4 centimeters or a head breadth greater than 16.1 centimeters, no target head was provided for development of a helmet of that size. In the 1990 survey sample, seven individuals have a head length or head breadth larger than the maximum and would not be accommodated if the largest target point was used for design of that particular size of helmet.



(Reproduced from Robinette and Whitestone, 1992)

Figure 3-9. Scatter Plot of Head Length and Head Breadth showing Target Values. Data is from 1990 U.S. Air Force Anthropometric Survey of Aviators.



(Reproduced from Robinette and Whitestone, 1992)

Figure 3-10. Scatter Plot of Head Length and Head Breadth showing Cut-off Lines for Helmet Sizes. Data is from 1990 U.S. Air Force Anthropometric Survey of Aviators.

There are a few other problems with the sizing system described above. First, the target values were chosen to be spread out across the population but it is unknown whether the target values chosen are the most appropriate. No statistical or mathematical techniques were used to find the optimum groups in the data. The optimum groups would have the least amount of variability within the group. That is, the groups would be divided such that the variance between the head lengths and head breadths in a group would be minimized. Statistical methods of defining the groups are available so that the variance is minimized within the group relative to the between the group variance. These statistical methods are most commonly referred to as cluster analysis and are investigated and discussed in Chapter 5, Results and Discussion.

The second problem with sizing and design scheme discussed above is that the individual chosen to represent the size will have a different shaped head than other individuals in the group. This different shape could present fit problems. For example, if the individual chosen to represent the group has a shape that results in dimensions smaller than a majority of the group's, then a majority of individuals would be unable to fit in that size helmet and would need to be accommodated by a size larger, if one exists. Obtaining a size larger may then result in the helmet being too loose and cause stability problems. A method of using all the heads in the group for design of the helmets, rather than just one individual, has been investigated in this research project and is discussed in Chapter 5, Results and Discussion.

CHAPTER 4

4 Description of Computer Programs for Calculation of Vectors of the Head

This section describes the computer programs developed for the calculation of the vectors from a selected midpoint in the head. Three FORTRAN 77 programs were written to accomplish this task. The first program determines the midpoints of the Cartesian coordinate data of a scanned head. Three different methods were used to determine the midpoints of the head, which were described in Chapter 3. The second computer program uses the midpoint data to calculate the vectors from the midpoint to the surface points on the head. The vector data calculated includes the spherical direction angles and the distance between the selected midpoint and the surface point. The third computer program then selects the vector in each scanned head that represents the vector of interest and compiles this data for all the heads into a database for statistical analysis. Each of these programs is described below.

4.1 Program: MIDPOINTS

This computer program determines midpoints of the head based on three different methods. The first method, which relied on determination of the

point furthest from the glabella, was discarded because of problems with the program locating points that were not on the surface of the head (i.e., errant points from the scanning process). The other two midpoints determined from the program were used in the calculation of the vectors, and results from these two midpoints will be compared and discussed in the next chapter. The program also makes a number of other calculations including the spherical direction angles and distance from the midpoint to the glabella. Figure 4-1 is a graphical depiction of the steps and calculations conducted in MIDPOINTS. Appendix B provides tables of the output of various calculations that can be obtained from MIDPOINTS for the head scans obtained from the 1990 U.S. Air Force anthropological survey of aviators.

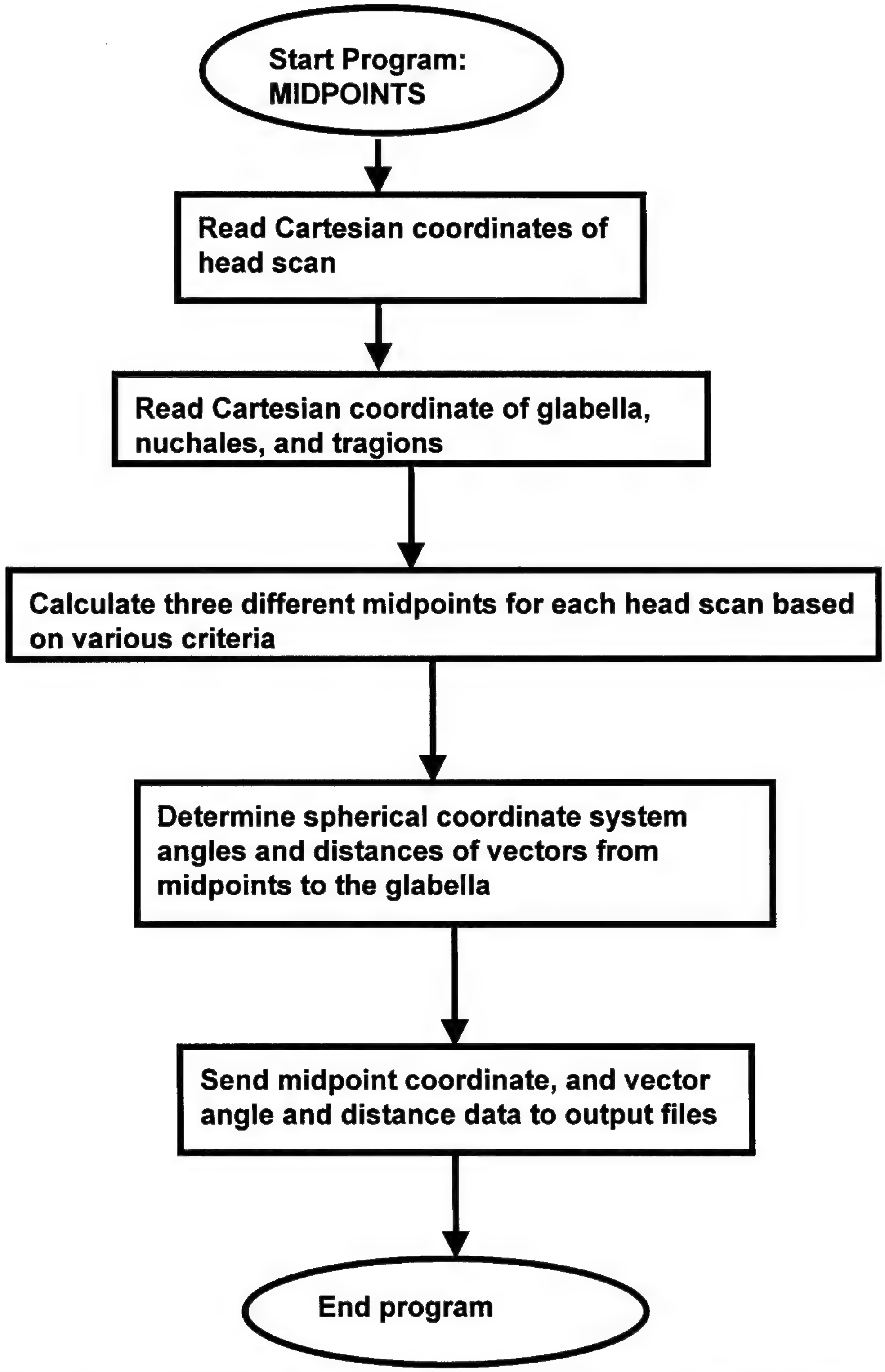


Figure 4-1. Summary of Steps in Computer Program MIDPOINTS

4.2 Program: VECTORS

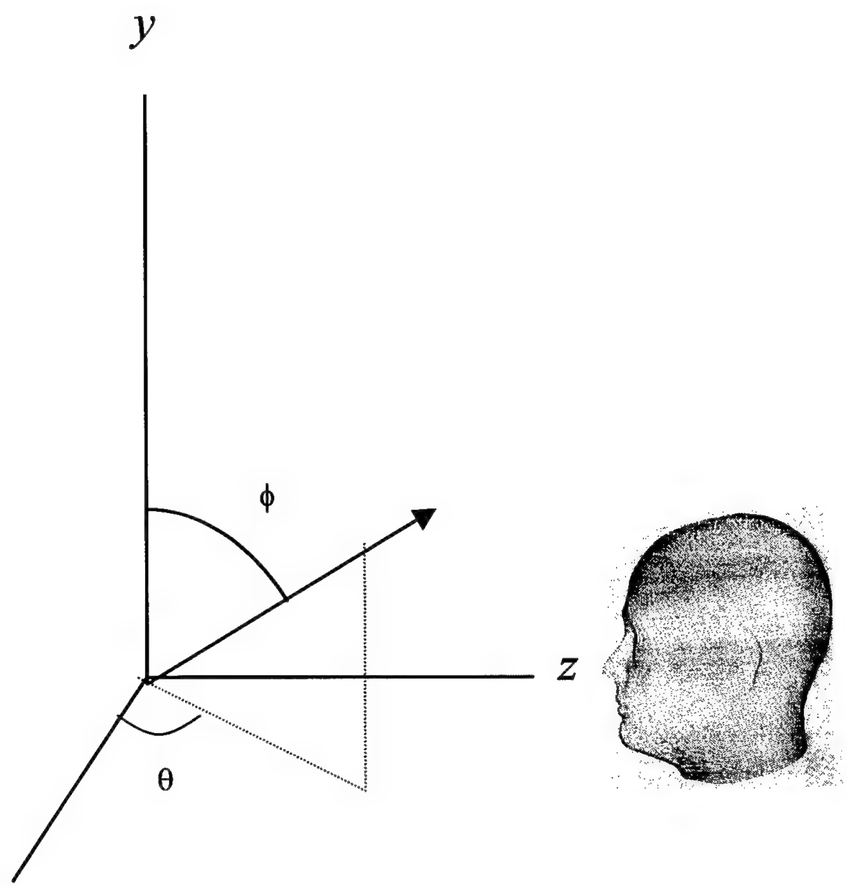
The second computer program uses the midpoints, determined in MIDPOINTS, to calculate the distance and spherical direction angles of the vectors represented by line segments that connect the midpoint and the points on the head's surface. In this program, the following equations are used to calculate the vectors' distance and direction angles:

$$d = [\ (x_s - x_m)^2 + (y_s - y_m)^2 + (z_s - z_m)^2\ ]^{½}$$

$$\phi = \arccos[(y_s - y_m)/\ d]$$

$$\theta = \arcsin[(z_s - z_m)/\ (d \times \sin\phi]$$

where the subscript s represents coordinate values for a point on the head's surface and the subscript m represents coordinate values of the midpoint. The term d equals the Euclidean distance from the midpoint to the surface point. The angle ϕ is the angle between the positive y-axis and the radial line segment with a distance of d. The angle θ is based on the projection onto the xz-plane of the line segment from the midpoint to the head's surface point. Figure 4-2 shows the spherical coordinate system used for the head scans and in the calculations of the vector's distance and direction angles. Figure 4-3 and 4-4 show two-dimensional and three-dimensional representations of the vectors that are calculated in VECTORS.



$$d = [\ (x_s - x_m)^2 + (y_s - y_m)^2 + (z_s - z_m)^2\ ]^{½}$$

$$\phi = \arccos[(y_s - y_m)/\ d]$$

$$\theta = \arcsin[(z_s - z_m)/\ (d \times \sin\phi)]$$

Figure 4-2. Spherical Coordinate System for Head Scan Data. Equations for direction angles are provided in the insert box.

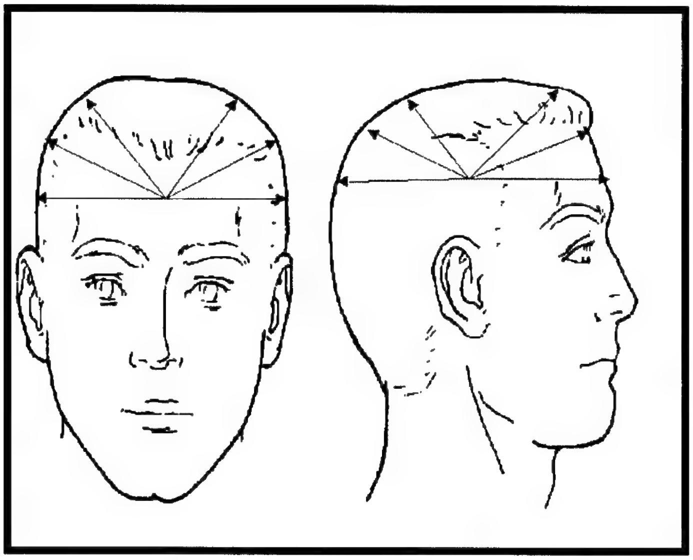


Figure 4-3. Two-dimensional Representation of Vectors Calculated in the Program VECTORS. Left head is front view. Right heade is side view.

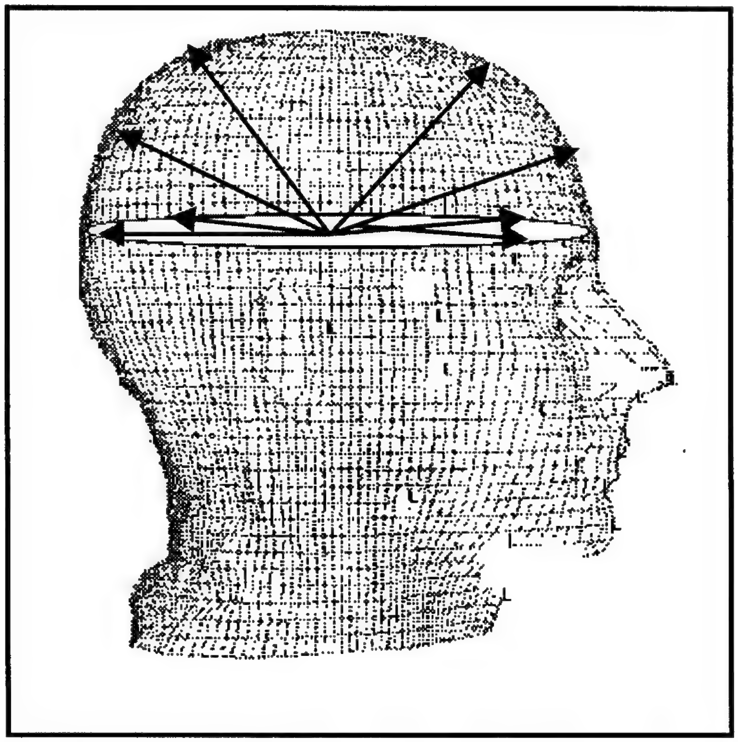


Figure 4-4. Three-dimensional Depiction of Representative Vectors calculated program VECTORS. Notice anatomical landmarks are identified with an L.

The computer program VECTORS writes out a file containing the distance and the associated direction angles for each point in the head scan data file. In this research, VECTORS was used to generate the distance and spherical direction angle files for heads scanned in the 1990 U.S. Air Force Male Aviator Anthropometric survey. Figure 4-5 provides a graphical algorithm detailing the steps and calculations performed by VECTORS.

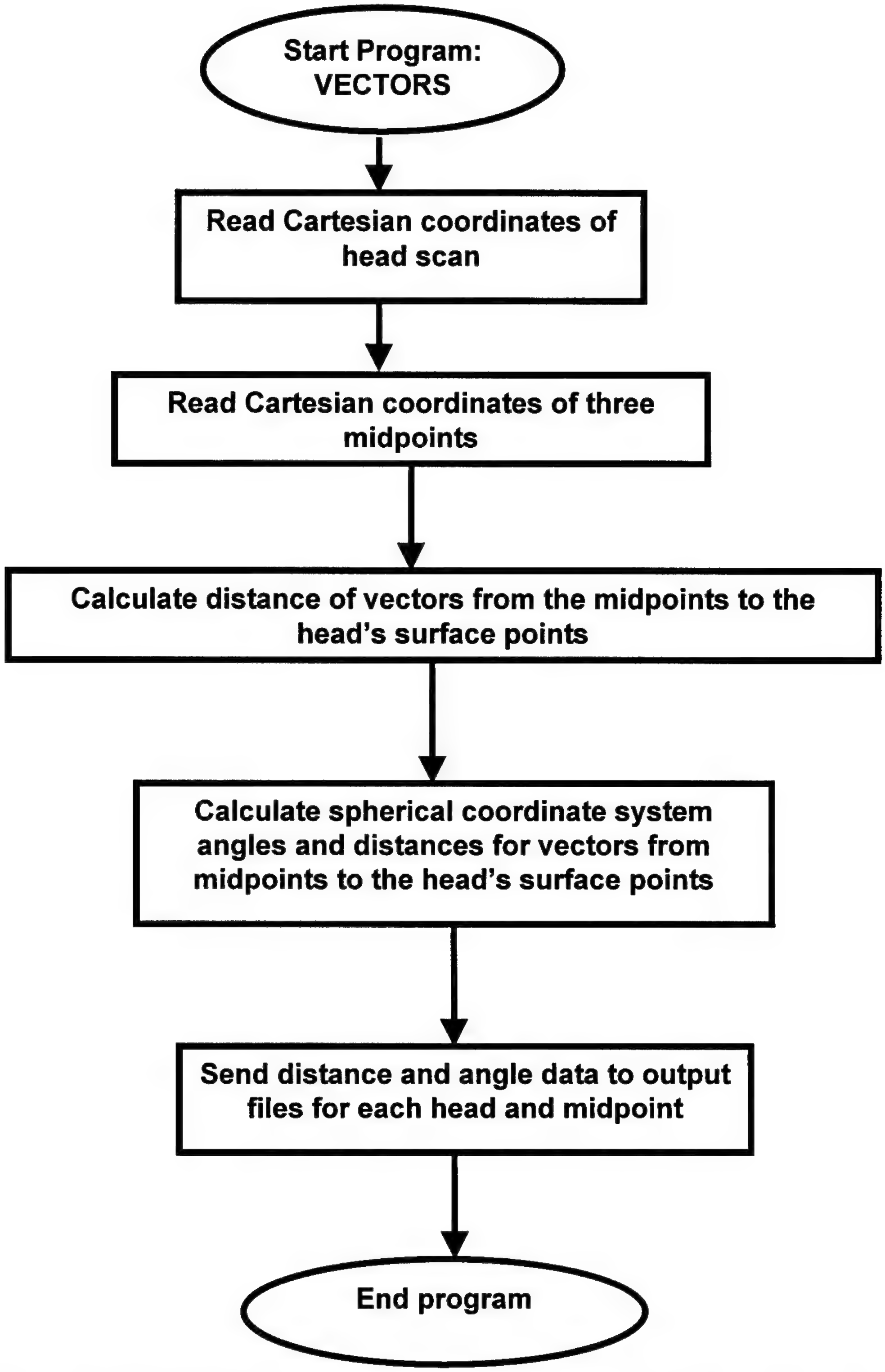


Figure 4-5. Summary of Steps in Computer Program VECTORS

4.3 Program: VECSELECT

The third Fortran program determines the vector from VECTOR's output file which most closely matches user-specified direction angles, and then writes to a file this vector's distance and filename. The program selects this vector by comparing the summation of the differences between the direction angles of interest and the direction angles for each vector in the file. The vector that has the smallest difference (usually less than a cumulative difference of 1 degree) is selected as the vector from the file which best represents the direction of interest. This value is then written to a file that will be used in statistical analysis of the vector data. Figure 4-6 is a graphical algorithm showing each of the steps in VECSELECT.

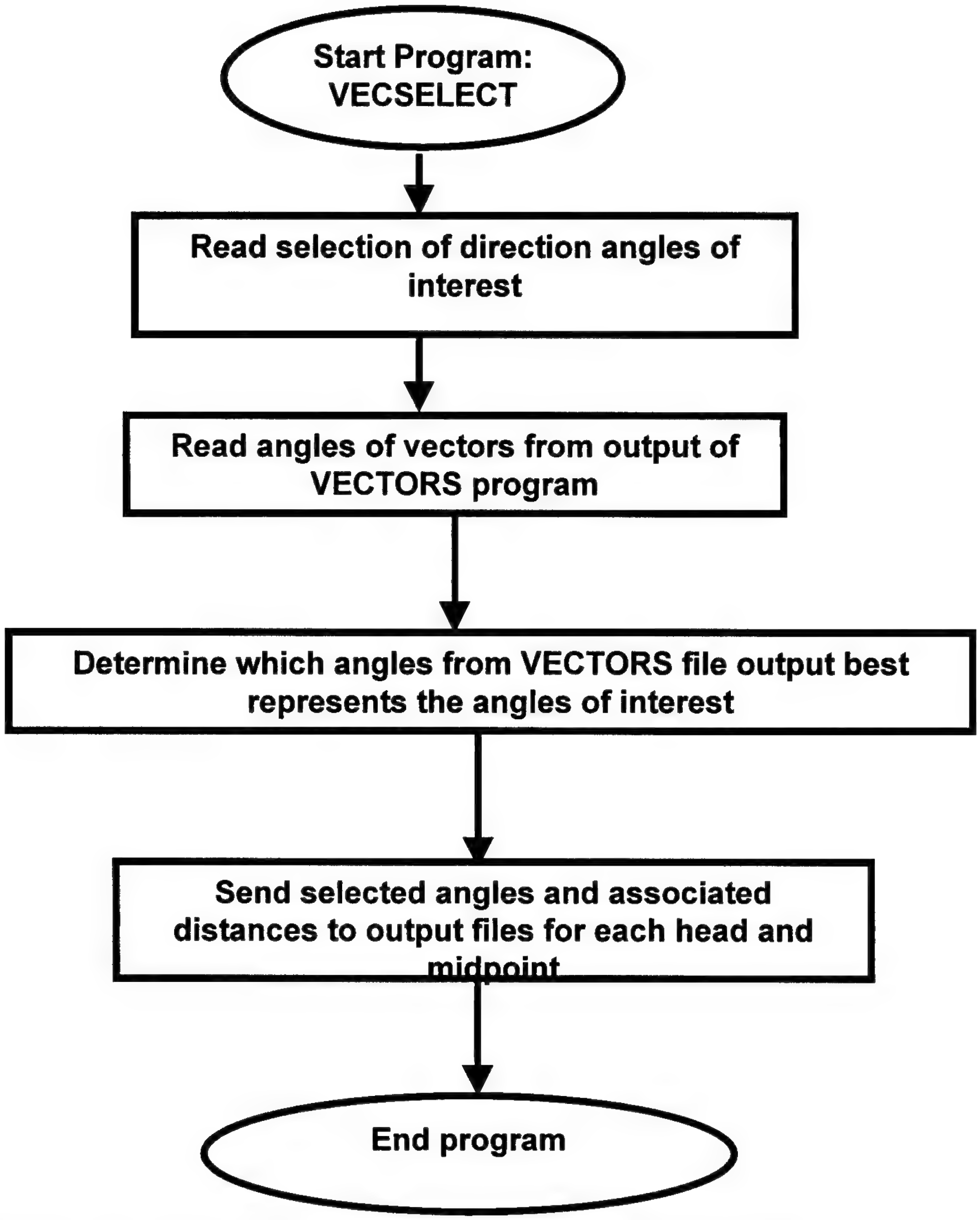


Figure 4-6. Summary of Steps in Program VECSELECT.

Chapter 5

5 Results and Discussion

This chapter provides results and discussion on the statistical analysis of the vectors and their use in the design of helmets. The first section compares the results of the two different midpoints to determine which method is better for helmet design. The second section discusses the methods investigated for sizing including Principal Component Analysis of the vectors and the use of clustering algorithms. The third section explores the methods for helmet design using regression analysis to predict the most appropriate and accommodating dimensions for the helmet. The fourth and last section of this chapter compares the present methodology used in helmet design to the use of the vectors and the regression analysis proposed in this dissertation.

5.1 Comparison of Methods for Determining the Midpoint of the Head

Three methods for determining the midpoint of the head were investigated in this research project. The first method, which relied on determining the most distant point from the glabella in the horizontal plane, was discarded because of problems with the program locating points not on the surface of the head (i.e., errant points from the scanning process most probably

due to light scattering problems). The second method determined the midpoint using the glabella and nuchales. The third method used glabella, nuchales, and left and right tragions for determining the midpoint. All of these methods were previously discussed in Chapter 3. Only the second and third methods were compared in this dissertation.

To compare the results, statistical analysis was conducted on the vectors resulting from the two midpoint methods. Using the programs VECTORS and VECSELECT, forty-eight different homologous vectors were first calculated and selected using each of the two midpoints for each of the head scans. Table 5-1 shows the spherical direction angles of these forty-eight vectors and their designated filename. The angles are presented as the difference from the angles from the reference vector (that is, $\Delta\theta = \theta_{(reference)} - \theta_{(of\ interest)}$; and $\Delta\phi = \phi_{(reference)} - \phi_{(of\ interest)}$).

Table 5-1: Vector Filenames and their Corresponding Direction Angles

Vector Filename	$\Delta\phi$	$\Delta\theta$	Vector Filename	$\Delta\phi$	$\Delta\theta$	Vector Filename	$\Delta\phi$	$\Delta\theta$
v00	0	0	v300	30	0	v600	60	0
v022	0	22.5	v3022	30	22.5	v6022	60	22.5
v045	0	45	v3045	30	45	v6045	60	45
v067	0	67.5	v3067	30	67.5	v6067	60	67.5
v090	0	90	v3090	30	90	v6090	60	90
v0112	0	112.5	v30112	30	112.5	v60112	60	112.5
v0135	0	135	v30135	30	135	v60135	60	135
v0156	0	156.5	v30156	30	156.5	v60156	60	156.5
v0189	0	180	v30180	30	180	v60180	60	180
vn022	0	-22.5	vn3022	30	-22.5	vn6022	60	-22.5
vn045	0	-45	vn3045	30	-45	vn6045	60	-45
vn067	0	-67.5	vn3067	30	-67.5	vn6067	60	-67.5
vn090	0	-90	vn3090	30	-90	vn6090	60	-90
vn0112	0	-112.5	vn30112	30	-112.5	vn60112	60	-112.5
vn0135	0	-135	vn30135	30	-135	vn60135	60	-135
vn0156	0	-156.5	vn30156	30	-156.5	vn60156	60	-156.5

The reference vector is from the midpoint to the glabella, and the reason for using this reference vector was described in Chapter 3, Basis for the Design and Sizing Methodology. For example, if the reference vector had a θ of 156° and a ϕ of 89°, then vectors at +/- 22.5° ° of the θ of 156°, and at less than 30° of ϕ, would be selected for each head. That is, two of the vectors for that particular head would be at {θ, ϕ}: {133.5°, 59°} and {178.5°, 59°}.

Since the head is approximately symmetrical with the dividing plane being the mid-sagittal plane, vectors determined from the midpoint should be approximately equal for two that have equal absolute values of $\Delta\theta$'s and equal

$\Delta\phi$'s, and for those pairs at $\Delta\theta=0°$ and $\Delta\theta=180°$ at equal $\Delta\phi$'s. For example, the magnitude of the vector at $\{\Delta\theta,\Delta\phi\} = (22.5, 30)$ should be approximately equal to the magnitude of the vector at $\{\Delta\theta,\Delta\phi\} = (-22.5, 30)$. Differences between the magnitudes of those to vectors would be due to asymmetry of the head, and whether the midpoint was exactly on the mid-sagittal plane. Therefore, an examination was conducted on the difference between corresponding vectors to get an indication if the midpoint is actually on the mid-sagittal plane. Then a statistical comparison was completed on the results of the different methods for determining the midpoint (that is, Midpoint Method 1 and Midpoint Method 2). Both midpoint methods should provide results with means of the differences of approximately zero. If the means turned out to be significantly greater than or less than zero, then the method was consistently determining the midpoint to be on one side of the mid-sagittal plane rather than on the other. In addition, the variance of the differences of the corresponding vectors should be approximately equal for the two methods. If one of the midpoint methods shows a significantly lower variance than the other method, then that method is more accurately placing the midpoint on the mid-sagittal plane than the other method. The method that shows the lowest variance would be considered the better method.

Appendix C provides the means of the differences for corresponding vectors and the results of two-sample t-tests for these means. The means of the differences for both methods were all less than +/- 3 mm which signifies that both methods were identifying midpoints fairly accurately on the mid-sagittal plane.

The two-sample t-test between the means did not indicate a significant difference between the two midpoint methods. The confidence intervals for the mean from Method 2 minus the mean from Method 3 included zero for all vector pairs. Figures 5-1 and 5-2 provide dot plots and box plots of the differences for the vector pairs at $\{\Delta\theta,\Delta\phi\}= \{0°,22.5°\}$. The t-test result for the vector pair at $\{\Delta\theta,\Delta\phi)= \{0°,22.5°\}$ is provided below (Diff022 is the difference between corresponding vectors at $\{\Delta\theta,\Delta\phi\}= \{0°,22.5°\}$):

<u>Two sample T for Diff022</u>

Midpoint Method	N	Mean	StDev	SE Mean
1	281	-0.35	2.36	0.14
2	281	-0.38	1.86	0.11

95% CI for mu (1) - mu (2): (-0.33, 0.38)

T-Test mu (1) = mu (2) (vs not =): T= 0.14 P=0.89 DF= 530

Although the t-tests on the means did not indicate difference between the two midpoint methods, a comparison test of the variances did find a statistically significant difference. Appendix F provides the results of the variance comparison tests for the two midpoint methods. Barlett's test for a normal distribution and Levene's test for any continuous distribution were used to compare the variances between the two methods. Both statistical tests consistently show that Midpoint Method 2 provides vectors with a statistically

significant lower variance for pair differences than Midpoint Method 1. This signifies that Method 2 is more accurately placing the midpoint on the mid-sagittal plane when compared to Method 1. Figures 5-3 and 5-4 are plots of the confidence intervals of the variances for the differences of the vector pairs at $\{\Delta\theta,\Delta\phi\}=\{0°,22.5°\}$ and at $\{\Delta\theta,\Delta\phi\}=\{0°,45°\}$.

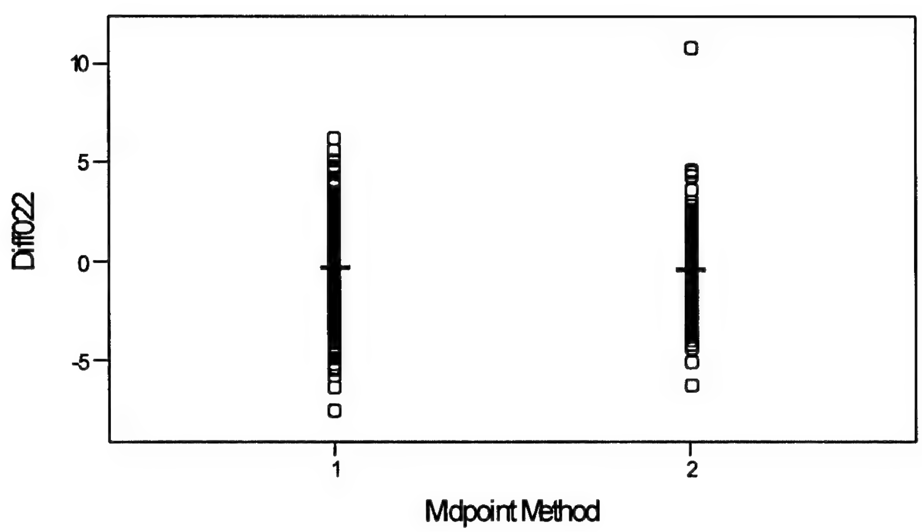


Figure 5-1. Dotplots of the Differences between v022 and vn022 for Midpoint Method 1 and 2. Means are indicated with red line.

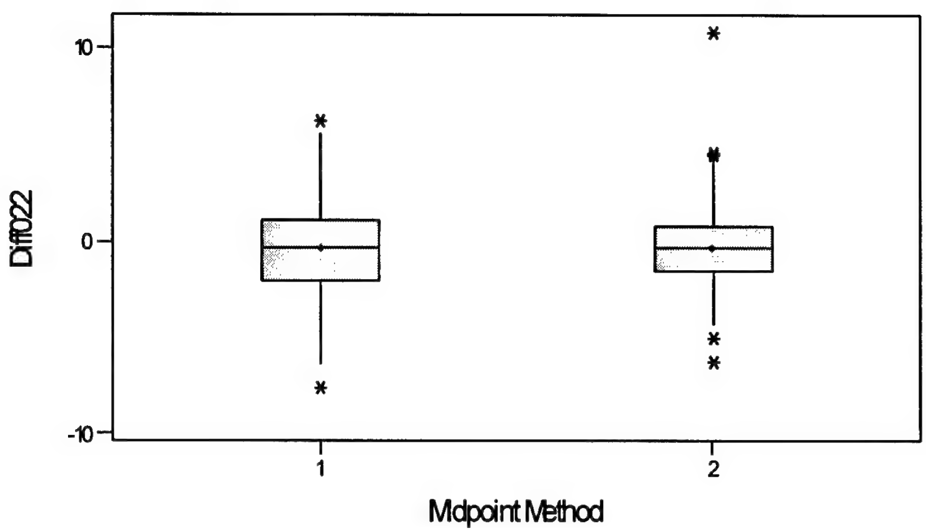


Figure 5-2. Boxplot of the Differences between v022 and vn022. The bottom of the box is at the first quartile (Q1) and the top is at the third quartile (Q3). The whiskers are the lines that extend from the top and bottom of the box to the lowest and highest observations still inside the region defined by the lower limit Q1 - 1.5 (Q3 - Q1) and the upper limit Q3 + 1.5 (Q3 - Q1). Outliers are plotted with asterisks (*). Solid red circle indicates mean.

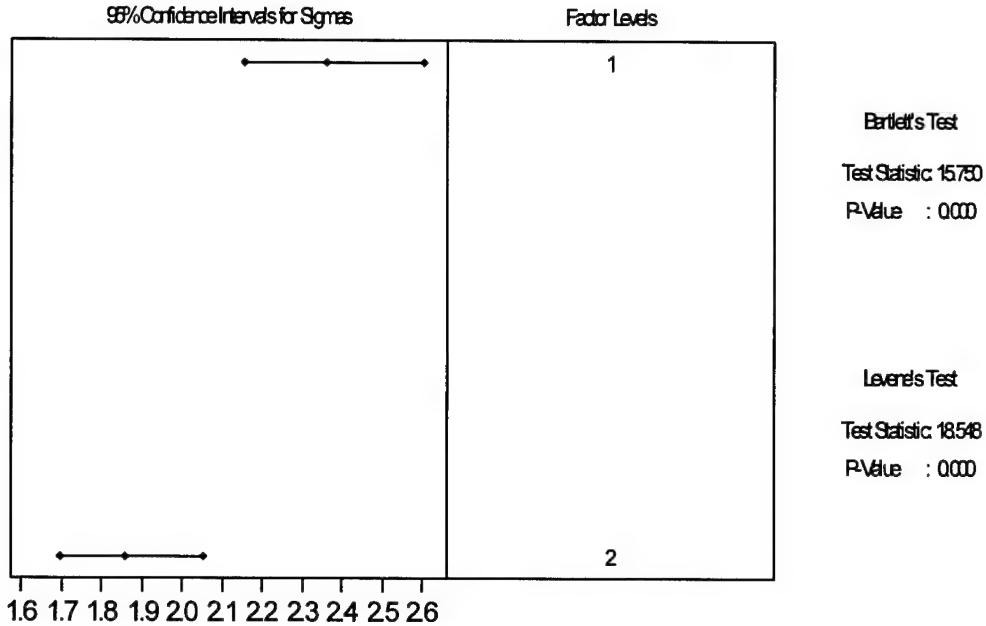


Figure 5-3. Confidence Intervals for Variances and Homogeneity of Variance Tests for the Difference between v022 and vn022 for Midpoint Method 1 and 2

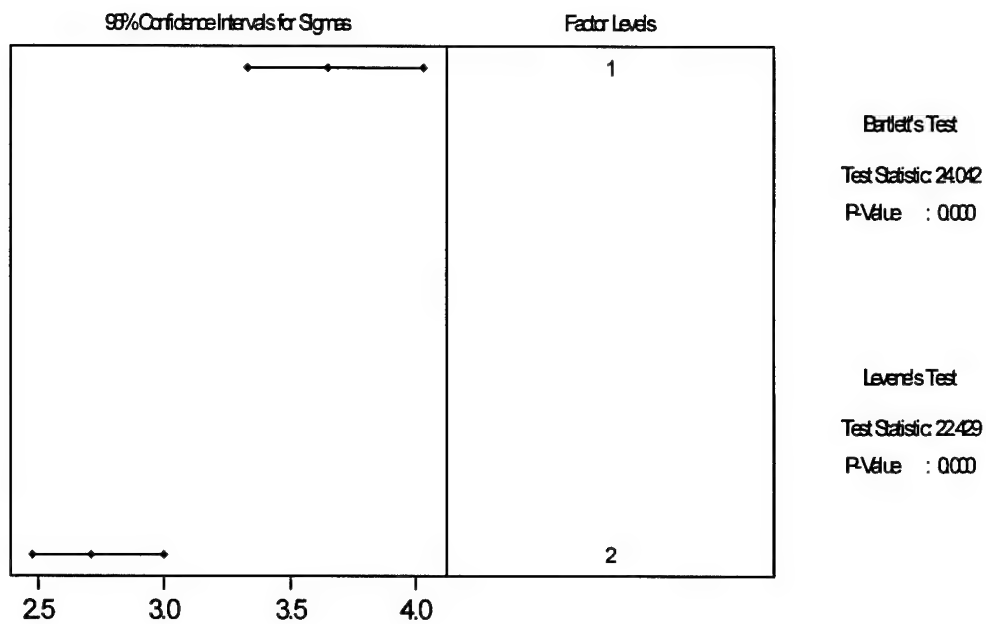


Figure 5-4. . Confidence Intervals for Variances and Homogeneity of Variance Tests for the Difference between v045 and vn045 for Midpoint Method 1 and 2

5.2 Evaluation of Methods of Sizing for Helmet Design

As discussed in Chapter 3, Basis for Design and Sizing Methodology, a currently used method for sizing of protective helmets includes developing a bivariate plot of head length and head breadth from a sample of the population of interest. Then, target points are chosen on this plot in a fashion as to accommodate a large proportion of the population (see Figures 3-9 and 3-10). The target points are in essence cut-off values for head length and head breadth, that is, heads which have a length or breadth larger than the cut-off must be accommodated in a size larger if one exists (see Figure 3-10). Two problems with this method are that: 1) No mathematical or statistical methods were used to identify the optimum groups within the bivariate plot (i.e., where should the target values be chosen as to optimize fit?); and 2) The individuals chosen to represent the population at a specific target site will have a different shaped head than other individuals in that size which will cause problems with fit. This section will address the first problem, while the second problem will be addressed in the next sections.

This research investigated the use of vectors for sizing of the helmets. As described in the previous section on methods of determining midpoint, forty-eight evenly distributed vectors of the head were selected for analysis. For the analysis shown in this section, vectors calculated from Midpoint Method 2 were used; Midpoint Method 1 provided similar results. The 48 vectors were then

paired together and the average calculated for each pair to form 24 vectors to represent the shape and size of each head. The average vectors were named avgXXXX, where the XXXX values represent the direction angles (for example avg045 equals (v045 + vn045)/ 2. Principal component analysis (PCA) was investigated and applied to these 24 vectors to reduce the number of variables. The objective of PCA is to identify the smallest number of factors that together account for a large percentage of the total variance in the correlation matrix of the original variables. Appendix D provides the complete output from the PCA, which includes the eigenvalues (the variances of the principal components), the proportion and cumulative proportion of the total variance explained by each principal component, and the score coefficients for each principal component. To determine the appropriate number of eigenvectors to extract from the analysis, scree test results are provided in Figure 5-5. As can be seen in the graph, the eigenvalues drop off quickly over the first four eigenvectors (i.e., a steep visual descent) and then level off into a slowly but steadily decreasing pattern of eigenvalues. The four eigenvalues and corresponding eigenvectors in the steep descent were retained for further analysis, and the eigenvalues in the gradual descent, including the eigenvalue occurring in the transition from the steep to gradual descent, were dropped.

Figure 5-6 is the score plot for the first two eigenvectors. In this figure, the scores for the second principal component (y-axis) are plotted against the scores for the first principal component (x-axis). This plot is used to determine if

any pattern exists in the data. No pattern is evident from this plot. Similar plots of the other principal component's scores showed comparable results.

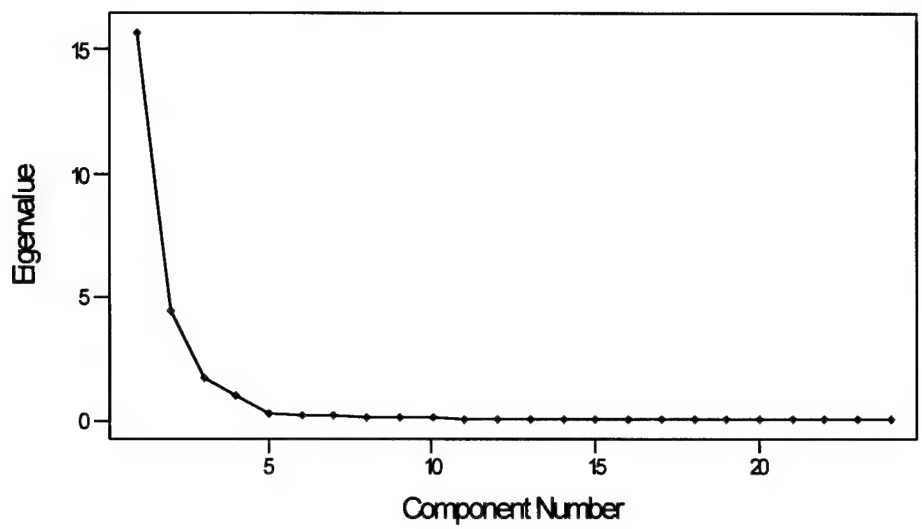


Figure 5-5. Scree Plot of Avg00 through Avg60157 (the Averages of the Corresponding Vectors)

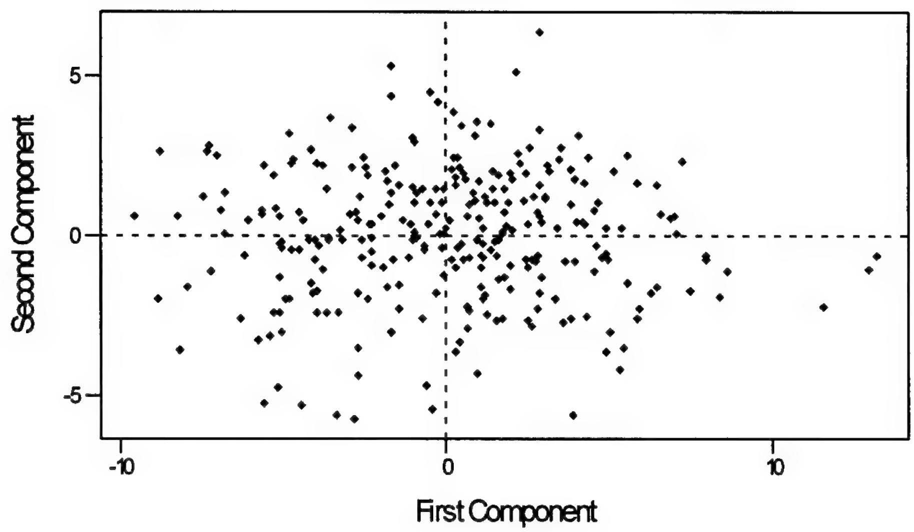


Figure 5-6. Score Plot for the First and Second Components of Avg00 through Avg60157

The next step was to use the four eigenvectors found from the PCA to develop a sizing scheme for our sample of aviators. Scores for each of the four eigenvectors were calculated, which were then used in a clustering routine for grouping into sizes. Principal component scores are specific values of an eigenvector calculated for a particular sampling unit (each head), and is formed as a weighted sum of the values of the variables (vectors) for that sampling unit. Appendix I shows the principal component scores for the sample of 281 aviator heads.

After reviewing a number of clustering algorithms and strategies, the K-means algorithm using MacQueen's algorithm (see Johnson and Wichern, 1988) seemed the most appropriate for sizing for helmet design. The purpose of K-means clustering is to classify observations into groups when the groups are initially unknown. K-means clustering begins with a grouping of observations into a pre-defined number of clusters. The method then evaluates each observation, moving it into the cluster whose centroid it is closest to, using Euclidean distance. When a cluster changes, by losing or gaining an observation, the program recalculates the cluster centroid. This process is repeated until no more observations can be moved into a different cluster, that is, until all observations are in the cluster whose centroid they are closest to. To begin the K-means algorithm, you must specify the initial groupings either by assigning values for the initial cluster centroids or by assigning an initial group for each observation. For the helmet sizing problem, an initial group for each

observation was designated. This was accomplished by first running an agglomerative hierarchical clustering routine for the four principal component score variables.

Agglomerative clustering begins with all observations separate, each forming its own cluster. In the first step, the two observations closest together are joined. In the next step, either a third observation joins the first two, or two other observations join into a different cluster. Each step results in one less cluster than the step before until, at the end, all cases are combined in one cluster. Once two observations are combined in a cluster, they may join with other observations, but they will always remain together. The cluster membership from the agglomerative cluster routine was then used as the initial grouping for the K-means routine.

To compare the results of the PCA and the clustering method to the current method of using target points on bivariate plot of head length and head breadth, a number of graphical and statistical methods were used. Figure 5-7 is a bivariate plot of head length and head breadth shown with different colors and symbols for the different sizing groups determined by the PCA scores and the clustering algorithms. Figure 5-8 is the same type of plot when target values are used to partition the sample into groups. Size 0 is for heads that would not fit into any helmet designed from the target heads. When comparing the two plots, one can see that the groups are much less compact and separated in

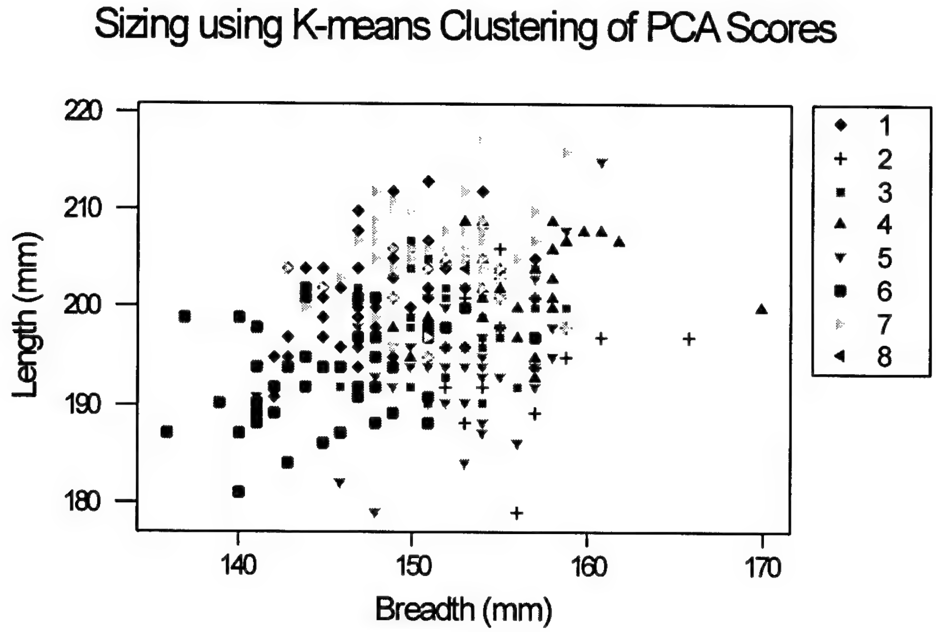


Figure 5-7. Bivariate Plot of Head Length and Head Breadth showing Sizes Developed from K-means Clustering of PCA Scores.

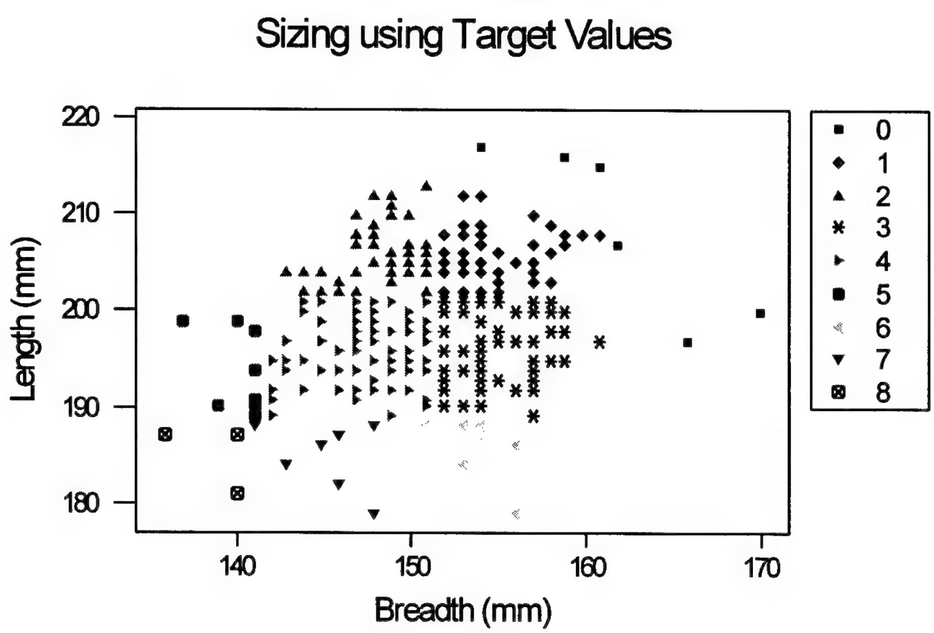


Figure 5-8. Bivariate Plot of Head Length and Head Breadth showing Sizes Developed using Target Values for Selecting Reference Heads

Figure 5-7 than in Figure 5-8, which indicates the method using PCA and K-means clustering did not work well for head length and head breadth.

Figure 5-9 is the same type of plot when all the vectors are used in the clustering routines rather than just the four PCA score variables. Again, this procedure provides clusters that are dispersed, which leads one to believe it does not work well for sizing of helmets. Figure 5-10 is same type of plot when the head lengths and head breadths are used as variables in the clustering algorithms. This plot shows a tight, compact grouping that is comparable to the target value method. Additional graphical comparisons can be seen in Figures 5-11 through 5-16. These figures provide dotplots of head length and head breadth by sizes for each of the methods. Notice that the dotplots for the PCA methods are much more spread out when compared to the K-means clustering of head length and head breadth, and the use of the target value strategy.

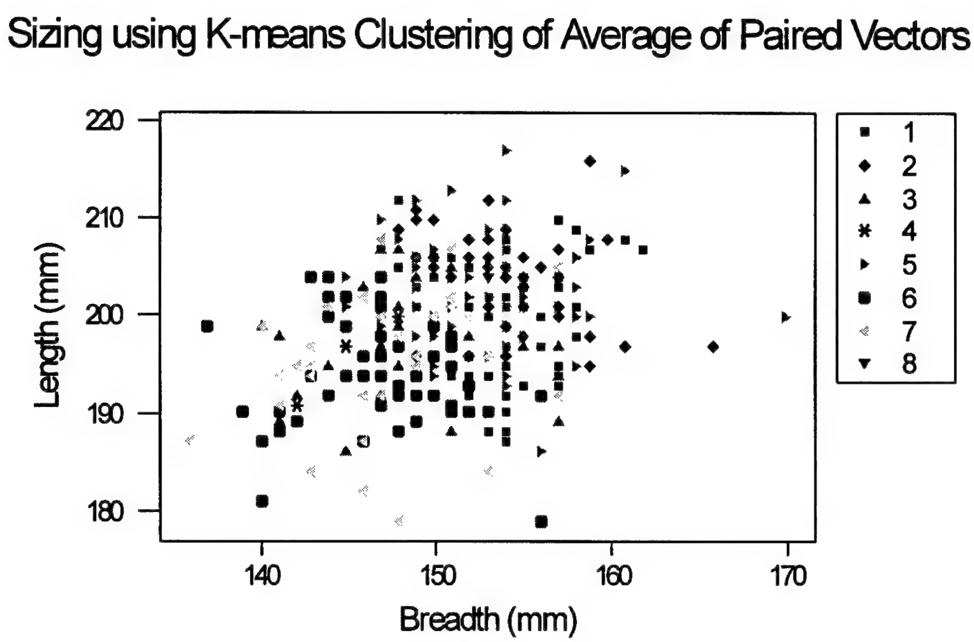


Figure 5-9. Bivariate Plot of Head Length and Head Breadth showing Sizes Developed from K-means Clustering of the Averages of the Paired Vectors (Avg00 through Avg60157).

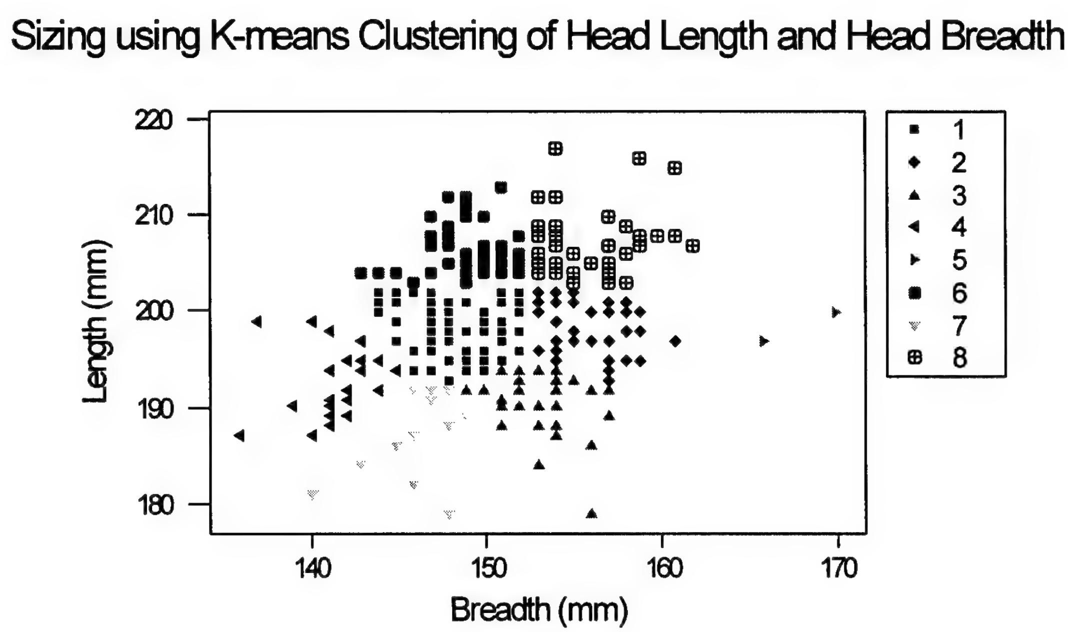


Figure 5-10. Bivariate Plot of Head Length and Head Breadth showing Sizes Developed from K-means Clustering of Head Length and Head Breadth

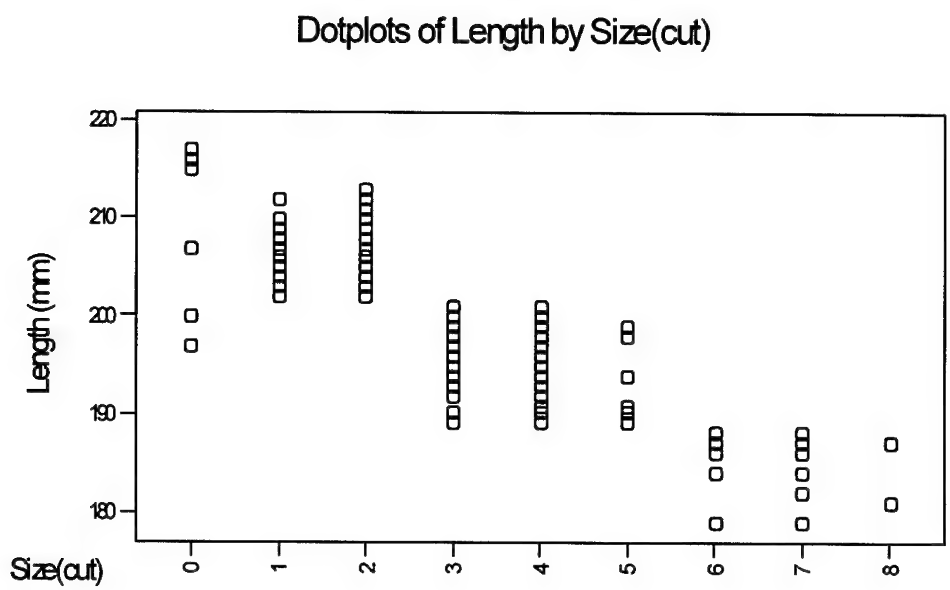


Figure 5-11. Dotplots of Head Length for Each of the Sizes Developed by the Target Value Method of Sizing

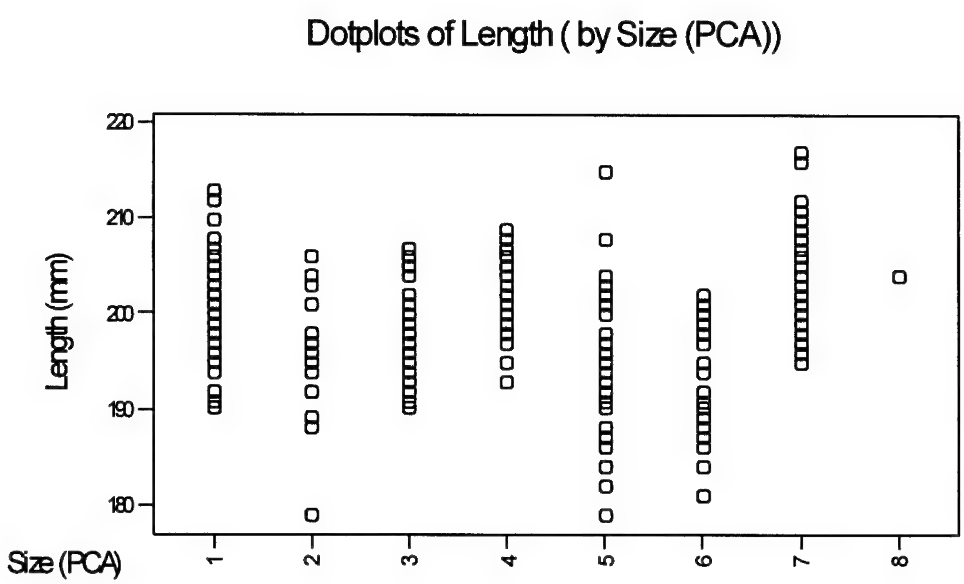


Figure 5-12. Dotplots of Head Length for Each of the Sizes Developed by K-means Clustering of PCA Scores

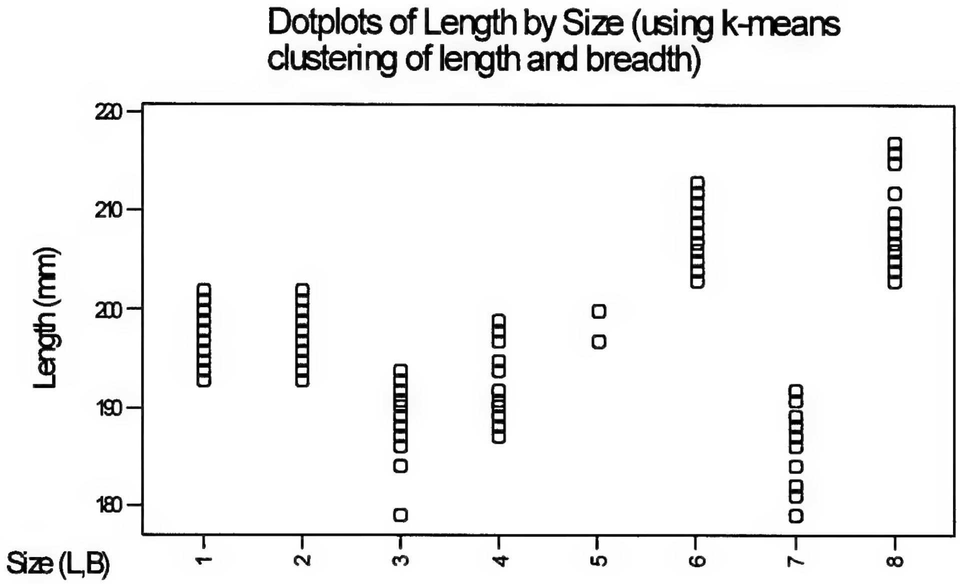


Figure 5-13. Dotplots of Head Length for Each of the Sizes Developed by K-means Clustering of Head Length and Head Breadth

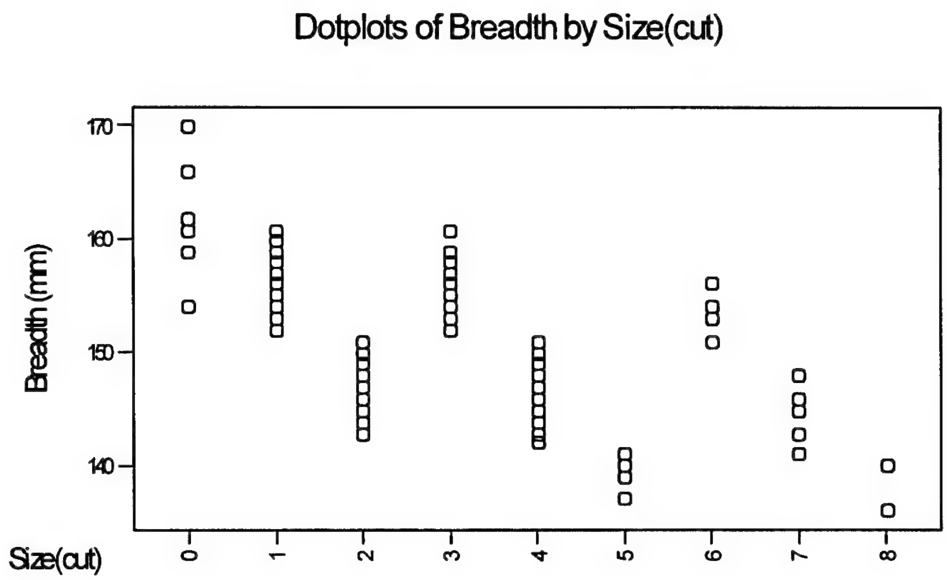


Figure 5-14. Dotplots of Head Breadth for Each of the Sizes Developed by Using the Target Value Method

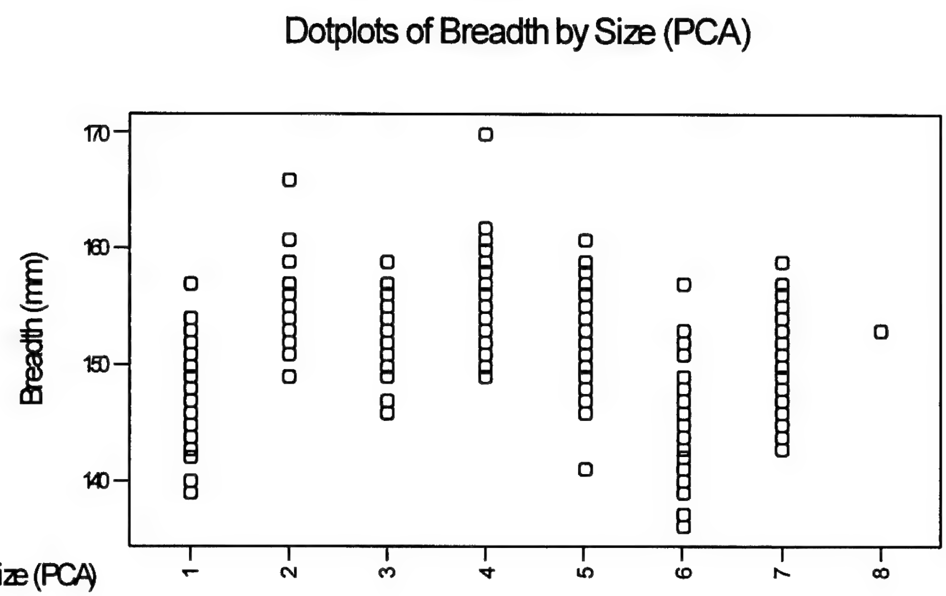


Figure 5-15. Dotplots of Head Breadth for Each of the Sizes Developed from K-means Clustering of PCA Scores

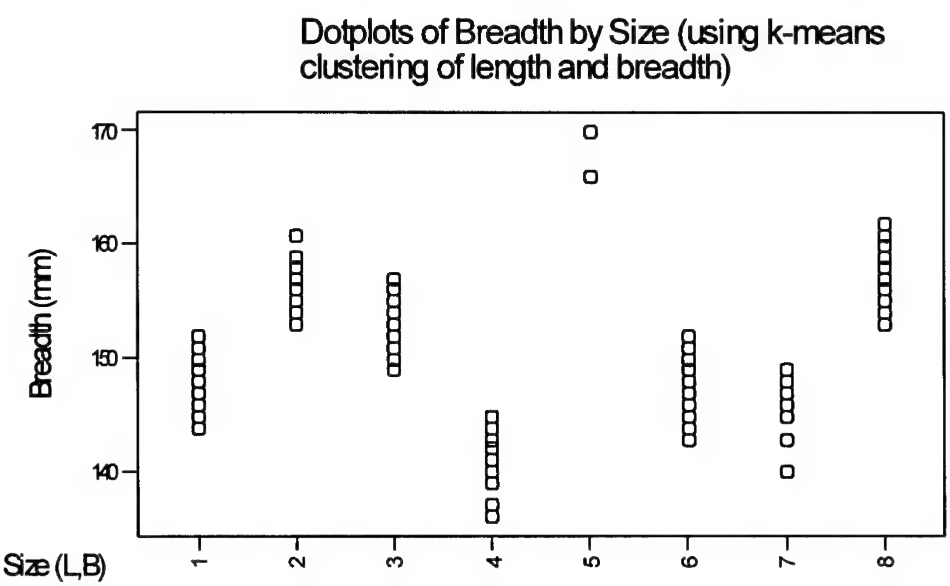


Figure 5-16. Dotplots of Head Breadth for Each of the Sizes Developed from K-means Clustering of Head Length and Head Breadth

The graphics presented so far have suggested that use of the PCA and clustering strategy may not provide a better method for sizing of protective helmets. To quantify the suggestive nature of the graphical presentations, a direct comparison of the gaps between the head and the helmets designed using the different sizing schemes was conducted. This was accomplished by subtracting the head length and head breadth of the design size formulated from the different methods from the actual head length and head breadth of the subject. These differences (i.e., deltas) would then be a quantitative measure of how well the helmet fit. A perfect fit would have a delta of zero.

One-way Analysis of Variance (ANOVA) was performed on the head length and head breadth deltas for the three different methods. The results of this analysis are provided in Appendix E. The results show that the PCA method had significantly greater differences between the helmet and the head than the other two methods. Two sample t-tests were then conducted on the deltas from the target value method and the K-means clustering of head length and head breadth method and the results are also provided in Appendix E. The t-tests reveal that the K-means clustering method provides significantly lower deltas ($p<0.10$) for both head length and head breadth. In addition, for the method using the target values, six individuals from our sample of heads would not be accommodated with a helmet (see Size 0 in Figure 5-8). For the other two methods, all individuals in the sample were accommodated.

Based on the analysis above, one might conclude that K-means clustering of head length and head breadth is the best method for sizing of protective headgear. However, in this analysis only two measurements, head length and head breadth, were compared between the different methods of grouping for sizes. Table 5-2 shows the variances for the vectors, head length, and head breadth for the three different methods of grouping individuals into sizes. The three methods, as described previously, are: 1) using target values (cut-off values), 2) using K-means clustering of head length and head breadth, and 3) using K-means clustering of PCA scores. F-tests were completed to compare the variances to one another and results of these tests are shown in Table 5-2 (with the exception of the F-test comparing the Cut Method to the K-means clustering of head length and head breadth method. All F-tests for each of the variables in this comparison were not significant at the 95% confidence level.) The method that works the best for sizing will have lower variances (that is, the spread of the vector's magnitudes will be lower). The variances were calculated by first determining the within size variance and then pooling these variances into one common estimate of the variance for a particular method. This common variance is estimated by the pooled variance equation found in many statistical books. The variances displayed in the table can be thought of as "size-weighted" variances for each of the three methods of grouping.

Notice from the table that the majority of variances for vectors that were determined by the grouping methods using the PCA scores were significantly

Table 5-2: Variances and F-tests for the Vectors, Head Length, and Head Breadth for Three Different Sizing Methods

Variable	Variances for Three Different Sizing Methods			F - test (K-Means of Length&Breadth/ K-means of PCA Scores)		F-Test (Target Valu Method/ K-means of PCA scores)	
	Target Value Method	K-Means Clustering of Length & Breadth	K-means Clustering of PCA Scores				
avg00	4.16	3.63	4.88	0.74	p<.05	0.85	NS
avg022	6.19	5.67	5.23	1.08	NS	1.18	NS
avg045	5.22	4.86	3.6	1.35	p<.05	1.45	p<.05
avg067	4.7	4.4	3.53	1.25	NS	1.33	p<.05
avg090	3.18	2.94	2.94	1	NS	1.08	NS
avg0112	2.43	2.16	2.44	0.88	NS	0.99	NS
avg0135	3.16	2.97	2.78	1.07	NS	1.14	NS
avg0157	4.67	4.2	4.56	0.92	NS	1.02	NS
avg300	25.8	25.3	15.6	1.62	p<.05	1.65	p<.05
avg3022	23.4	23.4	13.9	1.68	p<.05	1.68	p<.05
avg3045	16.6	17	9.56	1.78	p<.05	1.74	p<.05
avg3067	14.2	14.8	8.22	1.8	p<.05	1.73	p<.05
avg3090	13.3	13.8	7.97	1.74	p<.05	1.67	p<.05
avg30112	15.8	16.1	9.24	1.75	p<.05	1.71	p<.05
avg30135	21.7	21.7	12.8	1.69	p<.05	1.69	p<.05
avg30157	26.7	26.3	16.5	1.6	p<.05	1.62	p<.05
avg600	53	53.7	29.5	1.82	p<.05	1.8	p<.05
avg6022	50.5	51.6	27.2	1.9	p<.05	1.85	p<.05
avg645	50.8	52.4	26.9	1.95	p<.05	1.89	p<.05
avg6067	50.4	52.2	27.2	1.92	p<.05	1.85	p<.05
avg6090	55.7	58	30.1	1.92	p<.05	1.85	p<.05
avg60112	62.3	63.8	33.6	1.9	p<.05	1.85	p<.05
avg60135	65.3	66.2	36.8	1.8	p<.05	1.77	p<.05
avg60157	64.5	65.3	38.4	1.7	p<.05	1.68	p<.05
Breadth	5.7	5.12	14.6	0.35	p<.05	0.39	p<.05
Length	10.3	8.93	28.1	0.32	p<.05	0.36	p<.05

lower than the other two methods. The exception being only vector avg00 and head length and head breadth. This data provides evidence that grouping the heads by using the vectors, Principal Component Analysis, and K-means clustering can provide a better sizing methodology.

5.3 Development of Helmet Designs from Regression Analysis

In Chapter 3, problems with the sizing and design procedure of using target values on a bivariate plot were presented. One of these problems was that the individual chosen to represent the size would have a different shaped head than other individuals in the group. This different shape could present fit problems. For example, if the individual chosen to represent the group has a shape that results in dimensions smaller than a majority of the group's, then individuals with the larger dimensions would be unable to fit in that particular size helmet and would need to be accommodated by a size larger, if one exists. Obtaining a size larger may then result in the helmet being too loose resulting in stability problems. This section discusses an investigation of a method of using all the heads in the group and regression analysis for design of the helmets, rather than just one individual.

The proposed method for helmet design can be divided into the following steps: 1) Partition the entire sample into groups using the procedures described previously for sizing (that is, use K-means clustering algorithm); 2) For each group of subjects (that is, for each size helmet), conduct regression analysis using the vectors from the head as the dependent variables. For example, in

this research 24 average vectors have been calculated for each head; therefore, 24 equations for predicting vector magnitudes have be developed from regression analysis for each particular size helmet; 3) Using the regression equations, calculate the 99% prediction values for the vectors for each size helmet. Use these vector magnitudes to design the new helmet.

Before proceeding with discussion of the regression analysis, let us first explore and review the vector data. As described previously, forty-eight vectors were computed using the computer programs described earlier. These vectors were then put in a pairwise comparison to provide 24 vectors for description of the head. The first investigation completed was whether the vectors displayed a Guassian distribution. Tests for normality were conducted on all the vectors using the Anderson-Darling Normality test. The Anderson-Darling test is an empirical cumulative distribution function-based test. The null hypothesis of the test is that the data come from a normal distribution; the alternative hypothesis is that the data are not normal. A p-value less than 0.05 results in rejection of the null hypothesis; that is, to reject the hypothesis that the data are normal. Tests on the vector variables showed p values greater than 0.05 (that is, the vectors displayed a Guassian distribution) with the exception of only three. Figures 5-17and 5-18 show the results of the Anderson-Darling test for two of the vectors, avg00 and avg3090. Appendix J provides the normality tests for all the vectors along with other descriptive statistics for the vectors.

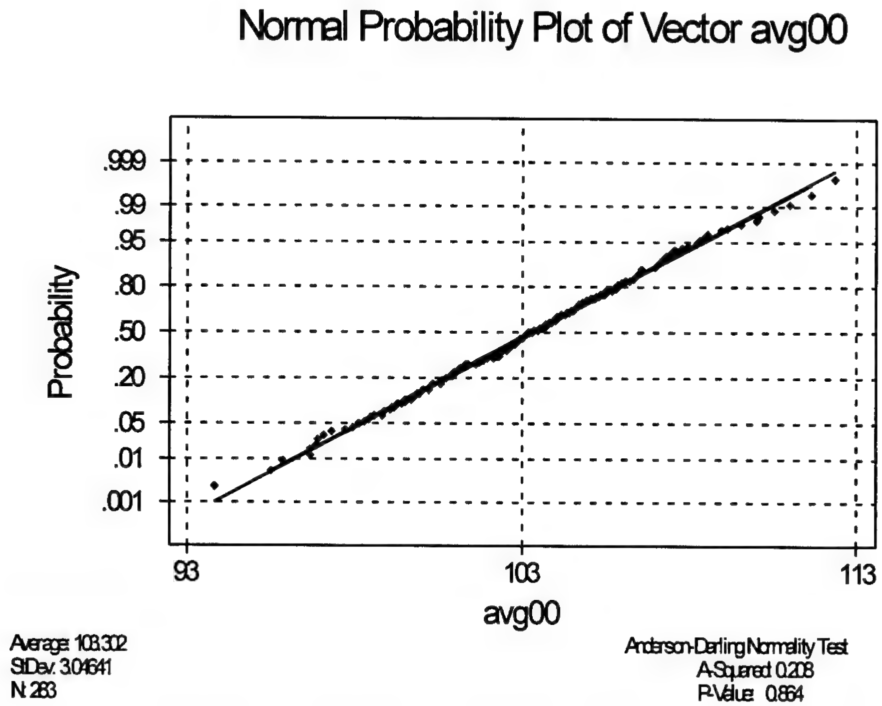


Figure 5-17. Normal Probability Plot and Test for Normality of Avg00

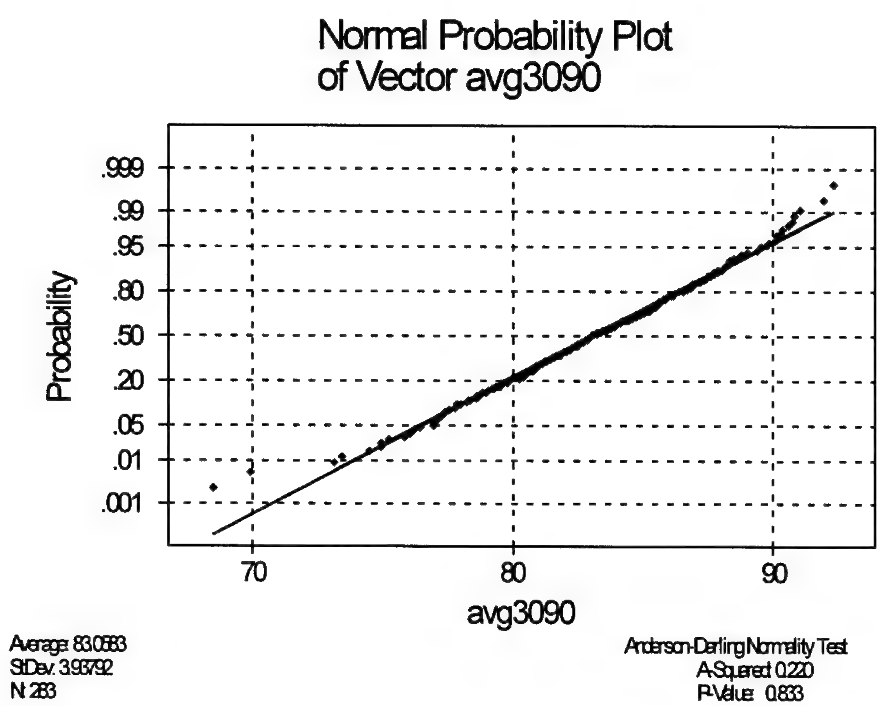


Figure 5-18. Normal Probability Plot and Test for Normality for Avg3090

Next, an investigation was conducted on the correlations between the vectors. Appendix G provides a complete correlation matrix for the vector variables found when using Midpoint Method 2. The correlation matrix for vectors from Midpoint Method 1 provided similar results. Figures 5-19 and 5-20 graphically depict results for two of the vectors. The graphs show that the correlations decrease as the direction angles between the two vectors increase up to a certain point, then the correlations begin to increase. The point where the correlations begin to increase is caused by the symmetry of the head. For example for v00 in Figure 5-19, the correlations decrease until approximately $\Delta\theta=90°$, then the correlations begin to increase. This is consistent with the low correlation between head length and head breadth, and is consistent with the results found from the PCA when reviewing the factor loadings after rotation (see Appendix D). For additional illustration, scatter plots of vector avg00 with avg022 and with avg090 can be found in Figures 5-21 and 5-22.

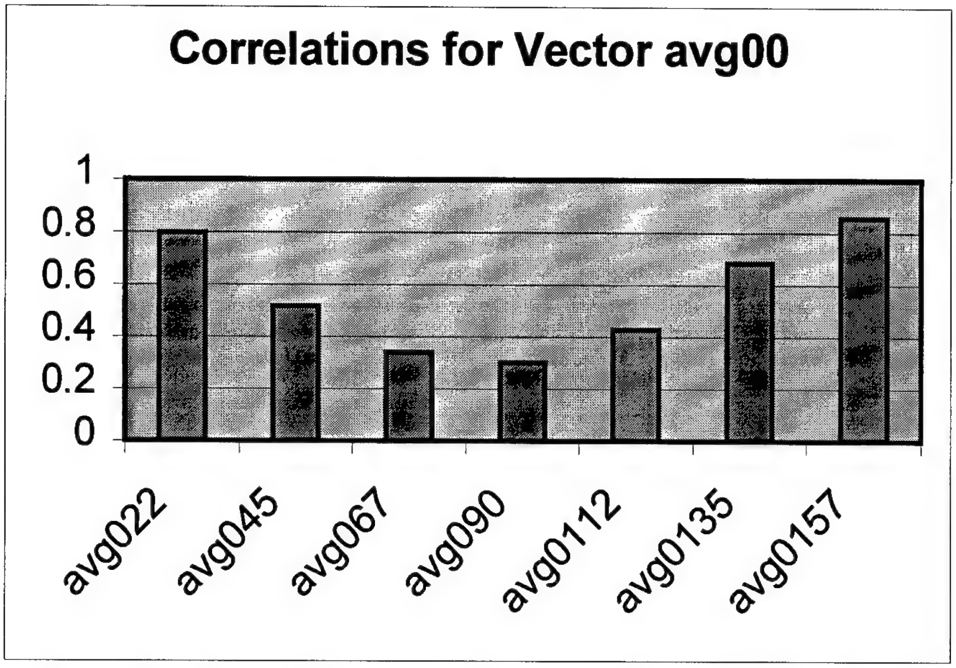


Figure 5-19. Bar Graph showing Correlation Coefficients for Avg00 with other Vectors

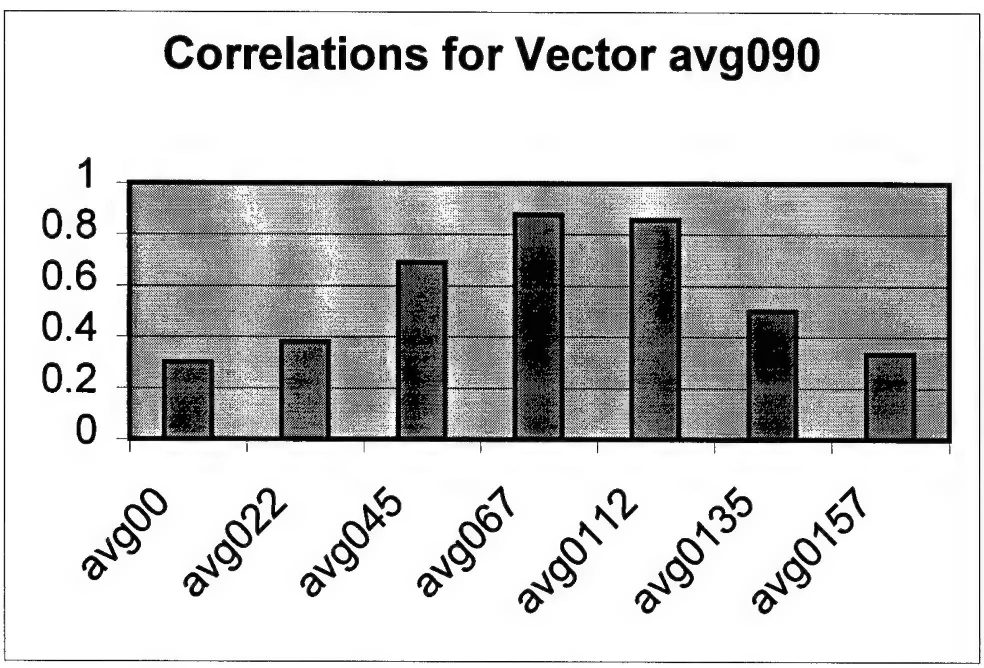


Figure 5-20. Bar Graph showing Correlation Coefficients of Avg090 with Other Vectors

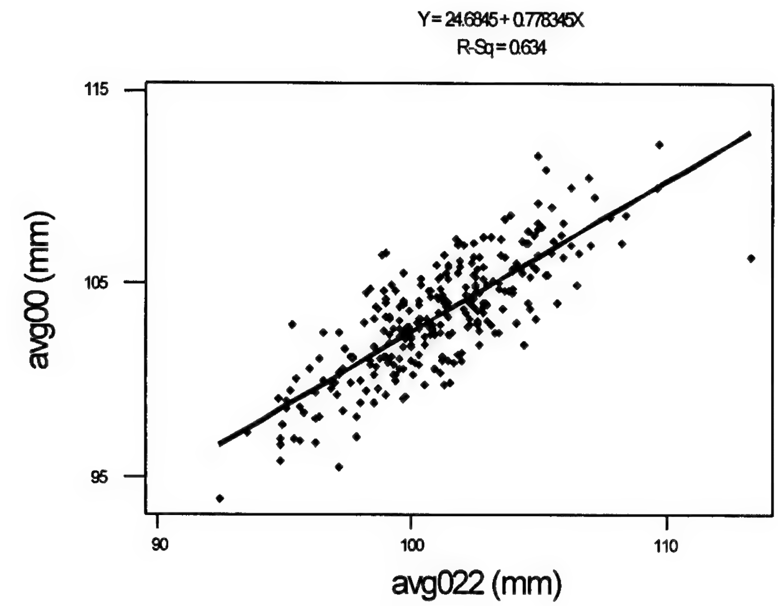


Figure 5-21. Scatter Plot with Regression Line of Avg00 and Avg022.

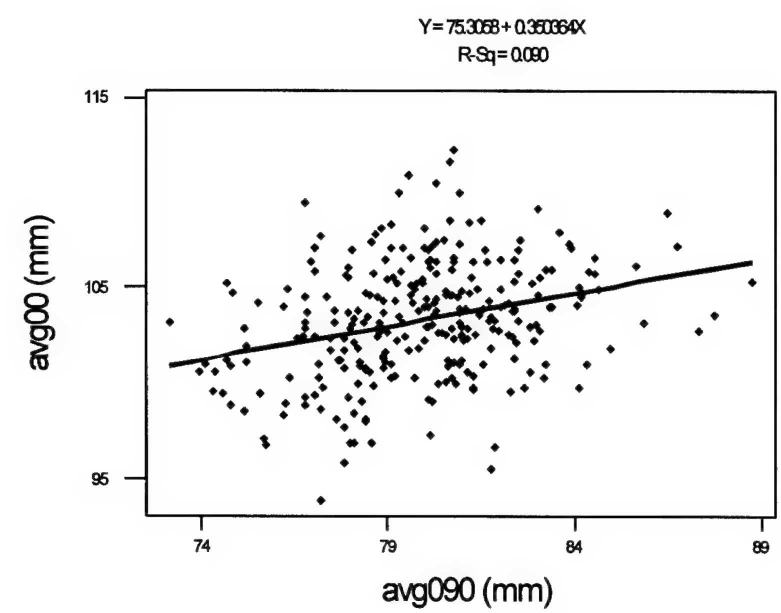


Figure 5-22. Scatter Plot with Regression Line of Avg00 and Avg090

The next task was an evaluation of the correlations between the traditional anthropometric measurements and the vector variables. Table 5-3 shows the correlations for head length, head breadth, head circumference, bitragion-coronal arc, and head height with the 24 vectors. Figure 5-23 is a graphical depiction of the data in this table. As would be expected, head length and head breadth are well correlated ($r > 0.80$) with vectors that are near its measurement points, such as v00 for head length and v090 for head breadth. As the distance from the measurement points increase, the correlation decreases. The measurement of Bitragion-Coronal Arc is more correlated with vectors resembling head length than those resembling head breadth, however the correlation never exceeds 0.561 for any vector. Head circumference is well correlated with vectors that are correlated with head length and only moderately correlated with those correlated with head breadth. Head height is poorly correlated with all vectors, however the correlation decreases as the vectors become more correlated with head breadth.

Table 5-3: Correlation Coefficients (Pearson's) of Traditional Anthropometric Measurements and Vectors

Vector	Height	Circ	B-C Arc	Breadth	Length
avg00	0.344	0.823	0.459	0.341	0.824
avg022	0.319	0.728	0.448	0.369	0.644
avg045	0.219	0.591	0.465	0.583	0.317
avg067	0.155	0.484	0.475	0.693	0.159
avg090	0.186	0.496	0.553	0.83	0.148
avg0112	0.197	0.636	0.561	0.826	0.348
avg0135	0.221	0.782	0.468	0.552	0.626
avg0157	0.253	0.81	0.407	0.372	0.753
avg300	0.39	0.376	0.474	0.226	0.307
avg3022	0.379	0.299	0.44	0.198	0.23
avg3045	0.375	0.28	0.49	0.305	0.133
avg3067	0.333	0.211	0.502	0.342	0.025
avg3090	0.322	0.208	0.495	0.347	0.027
avg30112	0.35	0.284	0.503	0.345	0.128
avg30135	0.372	0.355	0.497	0.303	0.231
avg30157	0.37	0.391	0.481	0.265	0.296
avg600	0.398	0.204	0.431	0.151	0.137
avg6022	0.378	0.141	0.405	0.118	0.069
avg6045	0.367	0.124	0.412	0.125	0.046
avg6067	0.353	0.126	0.413	0.129	0.05
avg6090	0.347	0.126	0.393	0.126	0.051
avg60112	0.359	0.16	0.416	0.146	0.08
avg60135	0.367	0.179	0.412	0.158	0.104
avg60157	0.377	0.208	0.424	0.169	0.136

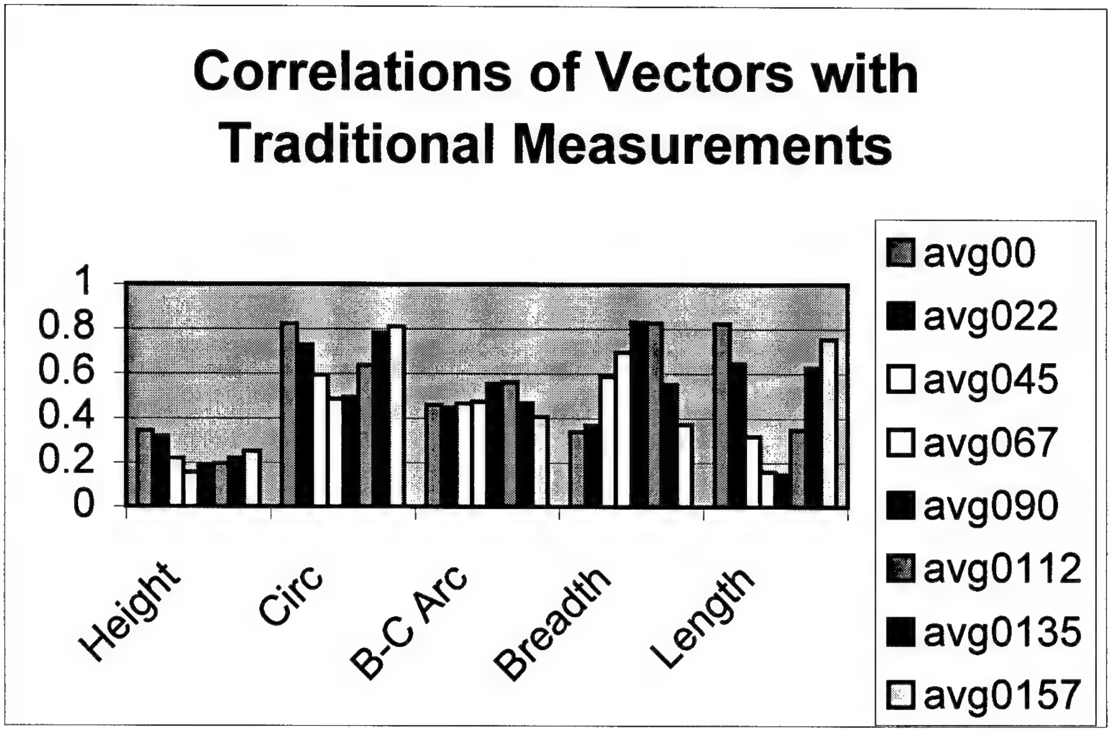


Figure 5-23. Correlation Coefficients (Pearson's) of Vectors (Avg00 through Avg0157) with Traditional Anthropometric Measurements

Let me now turn our attention to the regression analysis. Investigation was conducted to determine if the traditional variables would be good predictors of the vectors. A backward elimination procedure was used to determine the most appropriate regression equations using the five traditional measurements as the independent variables. The independent variables included head length, head breadth, head circumference, bitragion-coronol arc, and head height. The dependent variable was the vector; therefore, 24 regression equations for each size helmet were developed. In the backward elimination procedure, a regression equation with all the independent variables is first formulated. Then

the partial F statistic for every variable in the model is calculated as though it was the last variable entered. The partial F statistic tests whether the addition of the last variable to the model significantly helps in predicting the dependent variable given that the other variables are already in the model. Next, a comparison is completed between the lowest value of the F statistic with a pre-selected critical value of the F distribution. If the calculated F statistic is less than the critical value, the variable is removed from the equation and the entire procedure is repeated again until only variables which exceed the critical value are left in the equation.

Tables 5-4 and 5-5 provide results of the best-fit regression equations for Size 1 and Size 2 helmets, respectively, for the 24 vectors using the traditional measurements as the predictor variables. For Tables 5-4 and 5-5, sizes were determined by K-means clustering of head length and head breadth. Residual analysis and regression diagnostics were completed on the equations to determine if they met the assumptions of a linear regression model. Tests for the assumptions of linearity, homoscedasticity, and normality were conducted and example of these tests on a few of the vector equations can be found in Appendix H. All tests conducted showed no violations of the assumptions of linear regression analysis.

Results of the regression analysis provided in Tables 5-4 and 5-5 show that the traditional variables are poor predictors of the vectors. The multiple coefficient of determination, R^2, for a majority of the equations is less than 30

Table 5-4: Regression Equation Coefficients for Traditional Measurements from Vectors of Size 1. Sizes Determined by K-Means Clustering of Head Length and Head Breadth.

Vector	Const	Length	Breadth	Circ	B-C Arc	S	R-sq
avg00	49.45			0.093		1.53	15.08
avg022	30.08			0.123		2.04	14.7
avg045	60.71			0.174		1.93	25.1
avg067	79.98	-0.41		0.138		1.88	22.61
avg090	27.79	-2.66	0.323	0.083		1.46	38.91
avg0112	2.277		0.292	0.067		1.32	33.1
avg0135	20.86			0.126		1.42	27.47
avg0157	28.16			0.126		1.64	21.88
avg300	28.351				3.23	5.12	13.11
avg3022	35.21				0.17	4.69	11.44
avg3045	25.22				0.175	4.02	15.69
avg3067	12.39				0.199	3.9	20.39
avg3090	20.09				0.178	3.89	17.15
avg30112	24.56				0.18	4.23	15.07
avg30135	19.98				0.212	4.95	15.31
avg30157	25.75				0.206	5.23	13.21
avg600	-12.64				0.303	7.13	15.05
avg6022	-13				0.297	6.94	15.28
avg6045	-29.64				0.337	7.19	17.78
avg6067	-24.75				0.319	7.28	15.84
avg6090	-26.47				0.324	7.93	14.07
avg60112	-30.1				0.339	8.13	14.59
avg60135	-30.01				0.34	8.24	14.61
avg60157	-24.33				0.335	8.15	14.18

percent. The multiple coefficient of determination indicates what proportion of variation in the dependent variable is accounted for by all the independent (predictor) variables in the equation. In other words, R^2 gives an indication of how useful the multiple regression equation will be as a predictive model. If R^2 equals 100 percent, then the equation would provide a perfect prediction.

Table 5-5: Regression Equation Coefficients for Traditional Measurements from Vectors of Size 2. Sizes Determined by K-means Clustering of Head Length and Head Breadth.

Vector	Const	Lngth	Brdth	Hght	Circ	B-C Arc	S	R-sq
avg00	5.19	0.23			0.091		1.65	32.66
avg022	21.885				0.137		1.88	24.68
avg045	28.77	-0.47			0.263		2.06	42.11
avg067	-15.19	-0.34	0.31		0.2		1.82	45.65
avg090	-43.24		0.381	-0.095	0.105	0.049	1.26	52.07
avg0112	44.86				0.071		1.31	15.34
avg0135	34.16				0.104		1.5	22.56
avg0157	14.78				0.149		1.48	38.11
avg300	26.002					0.209	3.89	29.47
avg3022	22.906					0.206	3.69	31.18
avg3045	-58.77		0.53			0.182	3.06	41.88
avg3067	-48.15		0.46			0.17	2.81	41.94
avg3090	-34.46		0.43			0.145	2.92	33.91
avg30112	38.834					0.144	3.36	20.97
avg30135	36.32					0.169	3.89	21.55
avg30157	30.32					0.195	4.31	23.01
avg600	-26.96					0.344	5.85	33.4
avg6022	-25.093					0.33	5.57	33.85
avg6045	-23.19					0.318	5.41	33.46
avg6067	-24.33					0.317	5.61	31.78
avg6090	-21.85					0.311	5.86	29.07
avg60112	-36.62					0.358	6.35	31.58
avg60135	-30.59					0.347	6.77	27.67
avg60157	-26.86					0.342	6.73	27.34

Also notice in Tables 5-4 and 5-5 that most of the equations use either head circumference or the bitragion-coronal arc as the best predictors for the vectors. Head length and head breadth are rarely used and head height is used only one time for all the vectors in Size 1 and Size 2. Head circumference is

predominately used in equations where the vector's $\Delta\phi=0°$; bitragion-coronal arc is used for equations where the vector's $\Delta\phi=30°$and 60°.

Since the R^2 values for most of the regression equations in Tables 5-4 and 5-5 suggest poor predictions, other independent variables were explored for predicting the vectors' magnitudes. Tables 5-6 and 5-7 provide results of the best-fit regression equations for size 1 and size 2, respectively, for the 24 vectors using the principal component scores for the first four eigenvectors. For these tables, sizes were again determined by K-means clustering of head length and head breadth. The results suggest that the principal component scores can be extremely good predictors of the vector's magnitudes. Most R^2 values exceed 80 percent and the estimated standard deviation about the regression line for most of the vector equations is less than 1.0 mm. Residual analysis and regression diagnostics were also completed and showed no signs of violating the assumptions of a linear regression model (see Appendix H for examples of the tests).

Tables 5-8 and 5-9 are regression equation coefficients when the sizes were determined by K-means clustering of the PCA scores from the first four eigenvectors. These tables demonstrate, like the previous tables, that the traditional variables are poor predictors of the vectors, while the principal component scores are extremely good predictors.

Table 5-6: Regression Equation Coefficients for PC1-PC4 from Vectors of Size 1. Sizes Determined by K-means Clustering of Head Length and Head Breadth.

Vector	Const	PC1	PC2	PC3	PC4	S	R-sq
avg00	103.191	-0.341	0.73	1.075	-0.484	0.704	82.67
avg022	100.997	-0.443	0.882	0.635	-1.481	1.07	77.57
avg045	87.3914	-0.3717	0.9069	-0.5908	-1.2605	0.833	86.48
avg067	79.9288	-0.1672	0.9401	-1.2887	-0.5614	0.626	91.67
avg090	79.8444	-0.182	0.8798	-1.0104	0.5158	0.514	92.51
avg0112	84.4794	-0.1155	0.8164	-0.2688	1.0537	0.53	89.53
avg0135	93.4777	-0.1194	0.8922	0.7178	0.8159	0.5	91.34
avg0157	100.683	-0.304	0.982	1.262		0.636	88.6
avg300	98.9888	-1.3247	-0.1619	0.7699	-0.2748	0.89	97.49
avg3022	94.9917	-1.0899	-0.3572		-0.7183	1.71	88.54
avg3045	86.8257	-0.9846	-0.2326	-0.5236	-0.527	0.849	96.41
avg3067	82.4811	-0.9909		-0.7719	0.2408	0.719	97.37
avg3090	83.0436	-0.9576		-0.618	0.6582	0.851	96.15
avg30112	88.1925	-1.0996	0.2578		0.7171	0.9	96.28
avg30135	94.8968	-1.3192	0.3209	0.4631	0.6089	0.994	96.73
avg30157	98.3657	-1.3973	0.3184	0.9679		1.28	94.98
avg600	93.3978	-1.681	-0.9839			1.34	97.03
avg6022	90.9213	-1.5765	-1.2042		-0.61	1.72	94.96
avg6045	88.3426	-1.7021	-1.1237			1.34	97.17
avg6067	86.8824	-1.7084	-1.0696	-0.2684	0.3304	0.984	98.53
avg6090	87.0631	-1.8902	-0.9409		0.5201	1.04	89.57
avg60112	88.8017	-1.9459	-0.9826		0.3676	1.1	98.48
avg60135	90.6853	-1.989	-0.8265		0.5442	1.42	97.57
avg60157	93.1535	-1.9938	-0.5601			2	94.9

Table 5-7: Regression Equation Coefficients for PC1-PC4 from Vectors of Size 2. Sizes Determined by K-means Clustering of Head Length and Head Breadth.

Vector	Const	PC1	PC2	PC3	PC4	S	R-sq
avg00	103.478	-0.367	0.811	1.503	-0.654	0.949	78.71
avg022	101.591	-0.35	0.554	0.653	-1.68	0.871	84.75
avg045	86.8274	-0.3631	1.0173	-0.7911	-1.0665	0.789	91.89
avg067	79.6683	-0.2299	1	-1.2876	-0.374	0.576	94.69
avg090	80.2097	-0.1835	0.8264	-0.9551	0.3371	0.652	87.15
avg0112	85.002	-0.1158	0.8111		0.9585	0.45	90.46
avg0135	93.2463	-0.1565	0.9562	0.6868	0.7998	0.613	87.87
avg0157	100.433	-0.326	1.037	1.048		0.812	82.22
avg300	98.865	-1.3009		0.5757		1.05	95.01
avg3022	95.2661	-1.1524	-0.3427		-0.9391	1.61	87.45
avg3045	86.7976	-1.0495		-0.4953	-0.6362	1.19	91.35
avg3067	82.252	-0.9395		-0.7194		1.02	92.39
avg3090	82.948	-0.9085		-0.6155	0.6128	0.739	95.85
avg30112	88.3437	-0.9938			0.6795	0.884	94.65
avg30135	94.9047	-1.1939		0.3545	0.5932	0.826	96.61
avg30157	97.8185	-1.3786	0.344	0.5953	0.6413	0.905	96.81
avg600	93.9558	-1.8705	-0.7577	0.6786		1.96	92.84
avg6022	91.0925	-1.7278	-1.003			1.69	94.02
avg6045	88.7047	-1.6833	-1.0347			1.28	96.37
avg6067	87.2092	-1.7563	-0.9747			1	97.87
avg6090	86.8274	-1.7745	-0.9035	-0.4589	0.3671	1	98.04
avg60112	88.6331	-2.0134	-0.8839		0.313	0.9228	98.5
avg60135	90.8009	-2.0649	-0.8842		0.5112	1.339	97.3
avg60157	92.611	-2.0548	-0.7273		0.535	1.66	95.8

Table 5-8: Regression Equation Coefficients for Traditional Measurements from Vectors of Size 1. Sizes Determined from K-means Clustering of PCA Scores

VECTOR	CONST	LNGTH	BRDTH	CIRC	B-C ARC	S	R-SQ
avg00	23.38	0.284			0.066	1.61	59.67
avg022	46.51		-0.26	0.115	0.076	2	37
avg045	28.89			0.047	0.087	1.47	43.37
avg067	21.45		0.207		0.077	1.27	52.75
avg090	19.71		0.263		0.057	1.09	61.42
avg0112	24.3		0.296		0.046	0.946	69
avg0135	25.49			0.118		1.25	58.06
avg0157	10.777			0.157		1.81	53.82
avg300	38.71				0.161	3.8	14.61
avg3022							0
avg3045	49.42				0.095	2.89	9.4
avg3067							0
avg3090							0
avg30112	43.52				0.115	3.25	10.75
avg30135	34.78				0.16	3.92	13.67
avg30157	32.47				0.177	4.19	14.58
avg600	67.14		-0.42		0.236	4.01	21.16
avg6022							0
avg6045							0
avg6067	71.73		-0.45		0.214	4.41	16.48
avg6090	79.96		-0.52		0.219	4.8	16.01
avg60112	68.4		-0.48		0.253	4.45	20.46
avg60135	64.78		-0.46		0.248	4.86	17.04
avg60157	60.69		-0.48		0.276	5.19	17.92

Table 5-9: Regression Equation Coefficients for PC1-PC4 from Vectors of Size 1. Sizes Determined from K-means Clustering of PCA Scores.

Vector	Const	PC1	PC2	PC3	PC4	S	R-sq
avg00	103.1	-0.334	0.976	1.07	-0.73	0.742	91.79
avg022	101.1	-0.473	0.63	0.74	-1.66	1.03	83.72
avg045	87.46	-0.32	0.873	-0.85	-1.19	0.727	86.75
avg067	79.82	-0.157	1.043	-1.01	-0.25	0.573	90.63
avg090	79.57	-0.159	0.97	-0.95	0.37	0.514	91.81
avg0112	84.55	-0.119	0.899	-0.405	0.96	0.448	93.37
avg0135	93.23	-0.1	0.866	0.79	0.84	0.535	92.84
avg0157	100.6	-0.294	1.014	1.42		0.701	93.35
avg300	99.38	-1.359		0.38		0.872	95.61
avg3022	95.2	-1.219	-0.41		-0.81	1.45	85.31
avg3045	87.16	-1.07	-0.276	-0.54	-0.5	0.923	91.34
avg3067	82.7	-1.047	-0.198	-0.68		0.783	93.1
avg3090	83.74	-1.042	-0.195	-0.79	0.51	0.893	91.33
avg30112	88.81	-1.175		-0.4	0.85	0.866	93.94
avg30135	95.41	-1.394		0.35	0.54	0.862	96.01
avg30157	98.89	-1.455		0.62		1.2	93.15
avg600	92.54	-1.5546	-0.58			1.21	92.8
avg6022	89.94	-1.49	-0.72			1.63	86.91
avg6045	87.67	-1.623	-0.68			1.1	94.56
avg6067	86.62	-1.68	-0.766			1.01	95.66
avg6090	86.88	-1.819	-0.8			1.13	95.38
avg60112	87.86	-1.744	-64			0.872	96.94
avg60135	89.95	-1.799	-0.64	0.48	0.63	1.1	95.94
avg60157	93.08	-1.88	-0.45			2.11	86.45

5.4 Comparison of Helmet Design Methods

In section 5.3, regression analysis was explored for summarizing and predicting the vector data for design of helmets. Traditional anthropometric measurements were found to be poor predictors of the vectors needed for

design of the helmet, while the PCA scores were shown to be extremely good predictors. This section will discuss how these models can now be used for developing the design dimensions of a new prototype helmet and will compare this new method to the currently used procedure.

The sizing methodology explored suggested that K-means clustering of the PCA scores can provide a better means of grouping individuals than previously used methods. Using the K-means algorithm, the heads can be grouped so that the within group variance is minimized while the between group variance is maximized. After formulating the groups using these methods, the next task is to design an accommodating helmet for each group. To accommodate all the individuals in the group, the largest magnitudes for each of the vectors would need to be used. Using this method all individuals would be able to fit into the helmet (no helmet would be too small). The problem with using the largest magnitudes is that you risk the possibility of making the helmet too large, which could then cause stability problems and will increase the mass and moment of inertia of the helmet. If one individual in the group had a vector magnitude that was an outlier from the normal population, the helmet dimension designed from that magnitude could cause potential stability problems for the rest of the group. A better method would be to use a statistical technique such as using the 99 percent prediction values from the regression equations.

As described previously, Robinette and Whitestone (1992) used target values for head length and head breadth to design new prototype helmets.

Reference heads were selected near the target values for development of the design dimensions of new helmets. The problem with this procedure is that the shape and size of the reference head is different from the other heads in the group. When using the reference head as the design basis, head length and head breadth should be accommodated for a majority of the individuals in the group since the reference head has larger values for these dimensions than a majority of the group's. The problem, however, will be in the other dimensions of the head.

As an example of the potential problems with using a reference head, and to compare the design methods, Figure 5-24 shows the reference heads which would have been picked for the sizing system developed using K-means clustering of head length and head breadth. These reference heads were picked so that they are within 4 mm of the largest head length and head breadth values from the group. As an example, dotplots of the magnitudes of vector avg3090 for all sizes is shown in Figure 5-25. This figure shows the value for the reference vector in red. Notice that the magnitudes of the reference vector are always in the lower half of the all magnitudes. If the helmets were designed using the reference head vector magnitudes, the helmets would not accommodate a majority of the group for this vector. Other vectors investigated showed similar results. The design methodology proposed in this dissertation is to use the principal component regression equations to predict the most

appropriate design dimensions. It is proposed that the regression equations be used to calculate

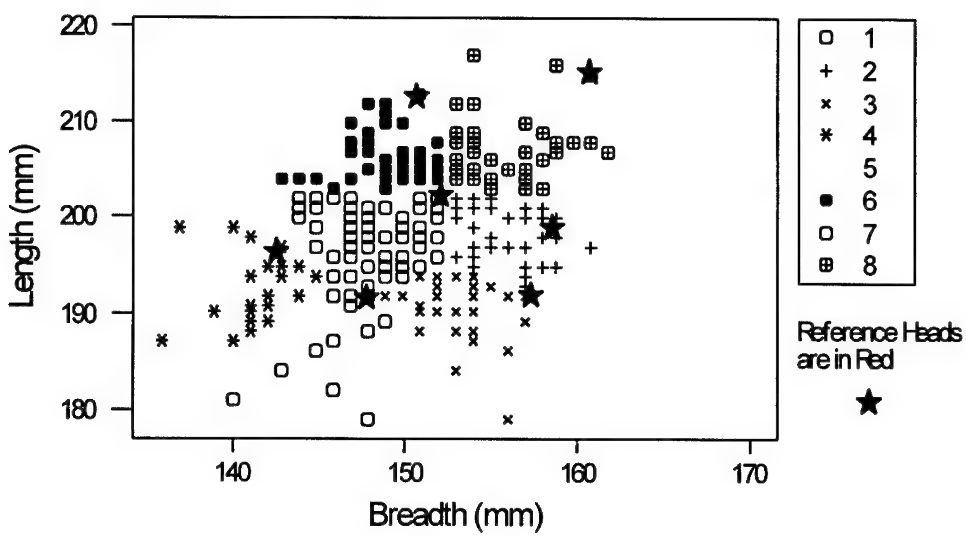


Figure 5-24. Scatter Plot of Head Length and Head Breadth showing Reference Heads for Each of the Eight Sizes. Reference Heads displayed with red Five Point Star.

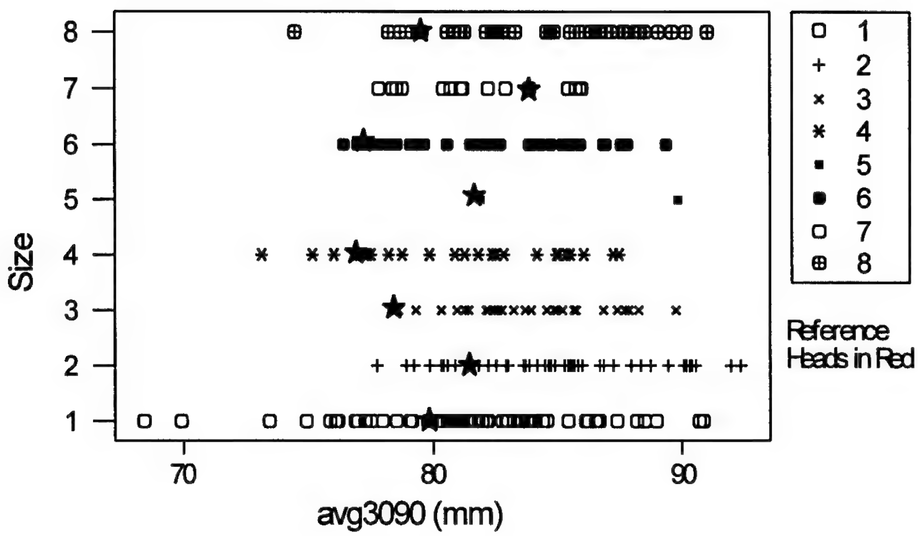


Figure 5-25. Dotplots of Vector Avg3090 showing Reference Head for Each Size displayed with Five-Point Star in Red.

the upper 99 percent prediction value for the observation with largest vector magnitude. Since the regression equations with the traditional measurement variables had relatively poor multiple coefficients of determination, the prediction intervals would be much larger than those from the principal component regression. The principal component regression equations provide relatively small prediction intervals. Therefore, it is recommended that the design dimensions for new helmets be determined by using the principal component regression equations and finding the upper 99 percent prediction value for the largest observation. By using these dimensions for design, a great majority of the population will be accommodated while compensating for outliers within the sample. As an example, Figure 5-26 shows the upper 99 percent prediction value for the vector avg3090 for all sizes using the principal component regression equations. Ninety-nine percent prediction values were also calculated using the regression equations with the traditional measurements as the predictor variables. As expected, the predictions were much larger than those from the principal component regression equations and would have resulted in unnecessary space between the helmet and the head.

The previous discussions have pertained to the vector data when the groups for sizes were determined by using the K-means clustering of head length and head breadth. As we have seen earlier, the variance within each size for the vectors was the smallest when K-means clustering was conducted on the

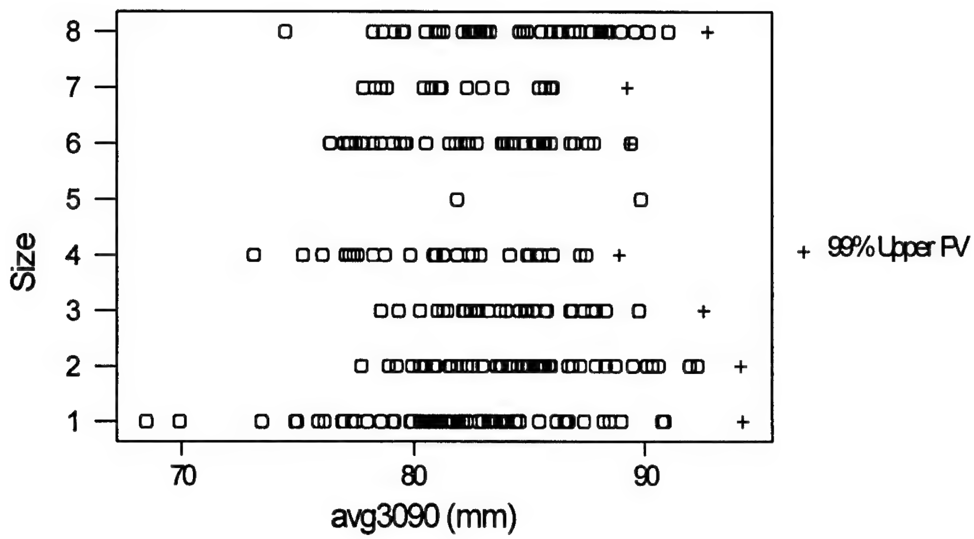


Figure 5-26. Dotplots of Vector Avg3090 showing Upper 99% Prediction Value for Largest Observation

PCA scores, rather than on the traditional variables of head length and head breadth. The variance for the head length and head breadth was, however, significantly larger when sizing by clustering of PCA scores compared to clustering of head length and head breadth, which is expected. The question is then whether the helmets designed using the sizing method of K-means clustering of PCA scores will accommodate a great majority of all the head lengths and head breadths in a specific size. The answer to this question can be explored by comparing the vectors nearest the measurements of head length and head breadth. For head length, the vector avg00 would be nearest; while for head breadth, the vector avg090 would be nearest. Multiplying these vectors by two and then comparing the values appropriately to head length and head

breadth should provide an indication of the accommodation of head length and head breadth by the vector design method. Figure 5-27 is a dotplot of values for each head equal to two times vector avg090 minus head breadth. Figure 5-28 is a dotplot of values for each head equal to two times vector avg00 minus head length. These plots show that in all but three cases both head breadth and head length could be accommodated by using the vector scheme for designing the helmets.

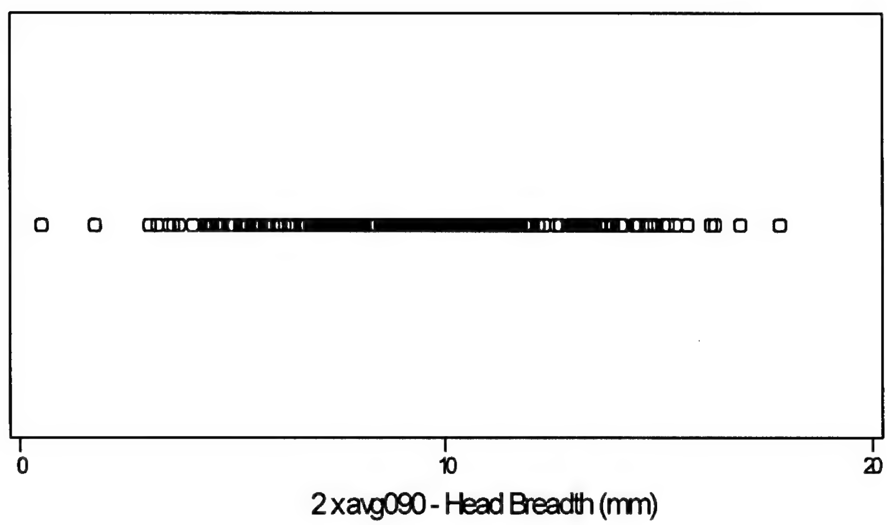


Figure 5-27. DotPlot of Two Times Vector Avg090 minus Head Breadth

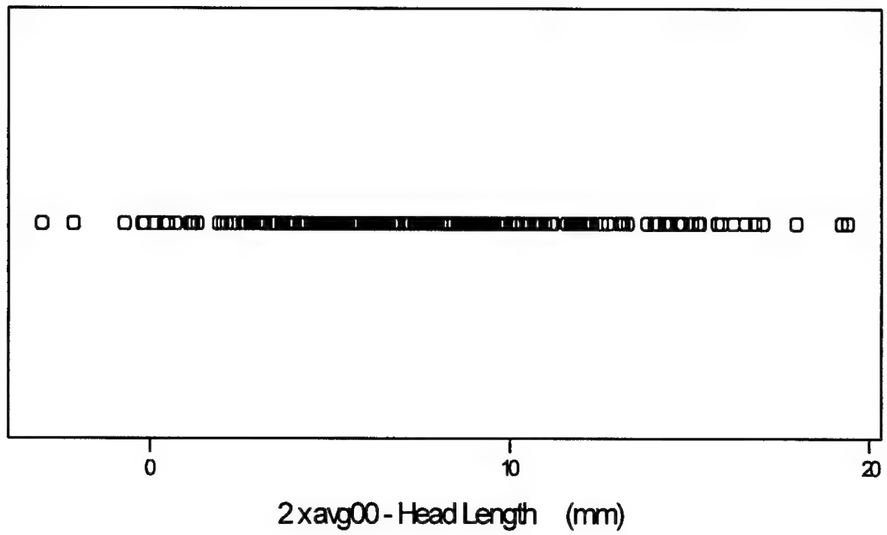


Figure 5-28. Dotplot of Two Times Vector Avg00 minus Head Length

CHAPTER 6

6 Summary and Conclusions

This dissertation has presented a new method for sizing and design of protective helmets using three-dimensional anthropometric data. The new sizing method includes conducting Principal Component Analysis (PCA) on vectors of the head. The vectors are found by defining a midpoint in the head and then calculating the distance and direction angles from the midpoint to the surface points. The scores from the PCA of the vectors for each of the heads are then used in a K-means clustering routine to partition the sample of heads into different sizes. The helmets are designed for each size based on regression analysis of the vectors and the PCA scores. The upper 99 percent prediction value for each vector is recommended as the design dimension for the helmet.

Three computer programs were written to determine the head's midpoint, calculate the vectors of the head, and select the vectors that are of interest for design of the helmets. The first program, named MIDPOINTS, determines the midpoints of the head based on selected criteria. The program also calculates a number of measurements, such as distance between landmarks and direction angles, using the three-dimensional anthropometric data from laser scanning

equipment. The second program, named VECTORS, calculates the distance (i.e., magnitudes) and direction angles of vectors in the head. The starting point of the vectors is the head's midpoint, determined from the program MIDPOINTS, and the ending points are the surface points on the head determined from the laser scanning equipment. The third program, named VECSELECT, uses the vector data calculated in VECTORS to select a vector of interest. The vector is found by comparing the direction angles of the vector of interest to the direction angles of each vector. The vector that has direction angles that are closest to the direction angles of interest is then selected and written to an output file for statistical analysis.

Figure 6-1 is a flowchart showing the steps in the new helmet sizing and design methodology developed in this dissertation.

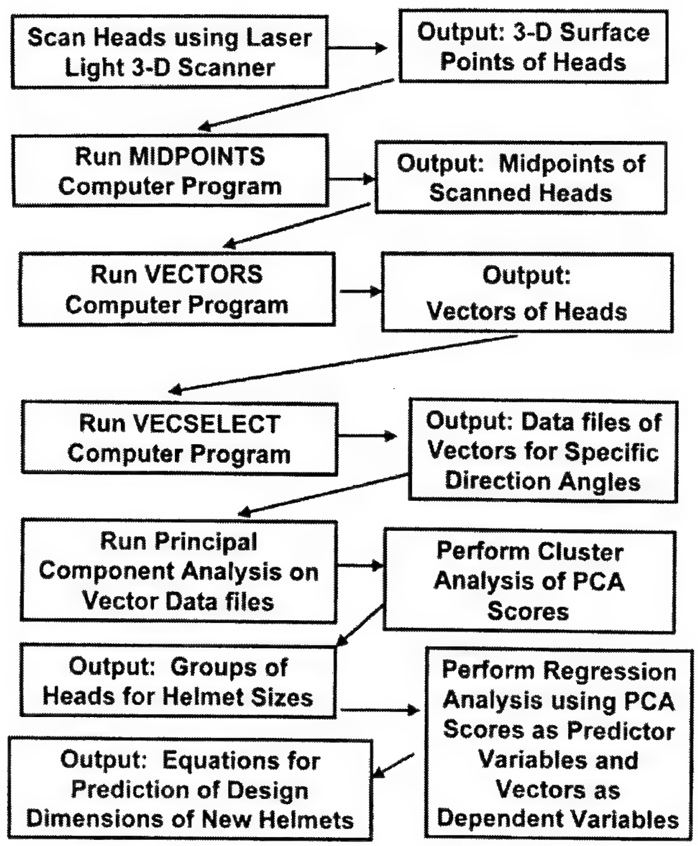


Figure 6-1. Steps in New Helmet Sizing and Design Methodology

Prior to selection of the vectors of interest, the vectors in each head must be placed into a common alignment system. For this research, the vector from the midpoint to the glabella was chosen as the reference vector to which all other vectors were compared. This alignment system was chosen because of design considerations for helmets. The opening for the eyes must always be below the glabella, and, for optimum viewing potential, the glabella should be vertically centered in the helmet opening.

This new method of sizing and design was compared to currently used method for design. The variances for the vectors were compared between the different sizing methods. Vector variances were significantly greater ($p<0.05$) for the sizing method of assigning a target value on a bivariate plot of head length and head breadth when compared to the variances from the method of clustering of the PCA scores. Reducing the variances will reduce the distances between the helmet and the head, providing a better fitting, lighter helmet. The new proposed design method of using the vectors in regression equations was also compared to the method of using the dimensions from a single reference head. The reference head was shown to provide helmet dimensions that would be unaccommodating; that is, the dimension proposed would be less than required for individuals in the size (i.e., the helmet would be too small). The regression equations developed using the PCA scores as the independent variables were found to be extremely good predictors ($R^2 > 80\%$) for the vectors needed for

design. The regression equations using the traditional measurements as the independent variables were found to be poor predictors of the vectors needed for design.

This new method of sizing and design could be used for any proposed number of sizes. The number of sizes is dependent on the performance criteria determined by the requirements for the helmet system and the costs of manufacturing and distribution. The larger the number of sizes the more close fitting the helmets will be to the head.

The new design method is dependent on determining the midpoint of the head and then calculating vectors from this midpoint. The number of vectors calculated from the midpoint equals the number of points found from the scanning process. However, for the Principal Component Analysis used in the design, the number of variables must be at least 10 times the number of subjects, as recommended by a number of statistical textbooks. The sample of heads used in this dissertation included 281 heads from the 1990 survey of Air Force male aviators. Therefore, the number of vectors used in principal component analysis was less than 28 (that is, 24 vectors were used in the PCA). For larger anthropometric samples, a larger number of vectors could be calculated and used in the analysis, as long as the ratio was less than or equal to 10.

Midpoints of the head for the calculation of the vectors were found by using the anatomical landmarks the glabella, the nuchales, and the right and left

tragions. A number of methods were used to find the midpoints and the results were compared. The method using all four of the landmarks was found to be the best. Future research should investigate methods of determining the midpoint that does not rely on anatomical landmarks, and attempts to exploit the symmetry of the head.

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APPENDIX A

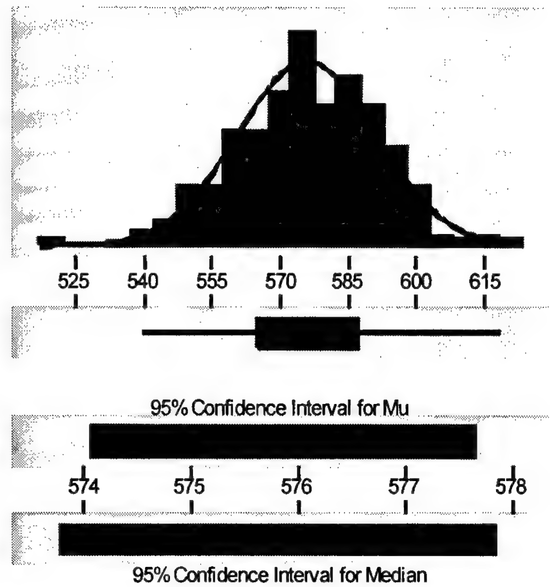
Descriptive Statistics of Traditional Anthropometric Measurements of Sample from 1990 U.S. Air Force Anthropometric Survey of Aviators

Descriptive Statistics

Variable	N	Mean	Median	Tr Mean	StDev	SE Mean
Circ	281	575.86	575.82	576.02	15.21	0.91
B-C Arc	281	348.71	347.98	348.57	13.10	0.78
Breadth	281	150.80	150.88	150.86	5.16	0.31
Length	281	198.87	198.88	198.95	6.75	0.40
Height	281	129.47	128.78	129.50	6.24	0.37

Variable	Min	Max	Q1	Q3
Circ	521.97	617.98	564.90	586.99
B-C Arc	319.79	385.83	339.85	358.90
Breadth	135.89	169.93	147.83	153.92
Length	178.82	216.92	193.80	203.96
Height	112.78	145.80	125.98	133.86

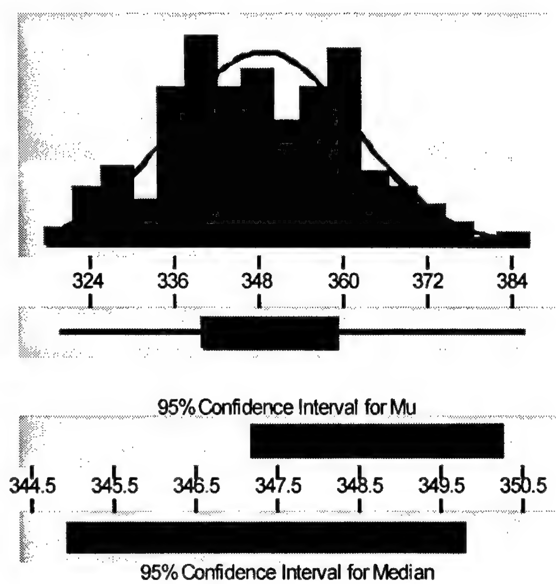
Descriptive Statistics



Variable: Circ

Anderson-Darling Normality Test	
A-Squared:	0.455
P-Value:	0.267
Mean	575.863
StDev	15.210
Variance	231.339
Skewness	-1.8E-01
Kurtosis	-1.5E-02
N	281
Minimum	521.970
1st Quartile	564.896
Median	575.818
3rd Quartile	586.994
Maximum	617.982
95% Confidence Interval for Mu	
574.077	577.649
95% Confidence Interval for Sigma	
14.048	16.583
95% Confidence Interval for Median	
573.786	577.850

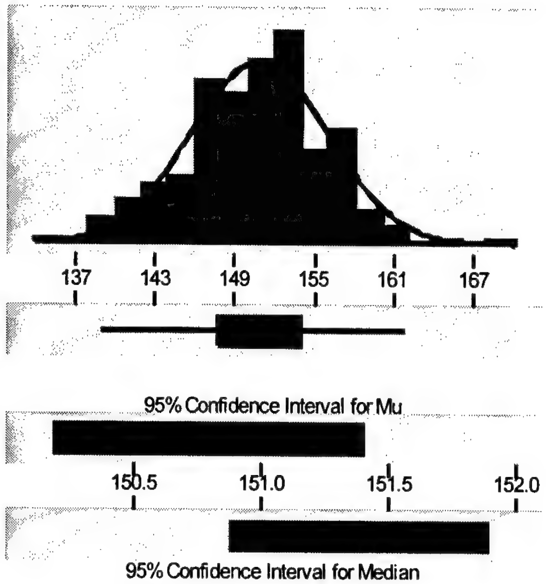
Descriptive Statistics



Variable: B-C Arc

Anderson-Darling Normality Test	
A-Squared:	0.862
P-Value:	0.027
Mean	348.707
StDev	13.095
Variance	171.486
Skewness	0.145339
Kurtosis	-4.5E-01
N	281
Minimum	319.786
1st Quartile	339.852
Median	347.980
3rd Quartile	358.902
Maximum	385.826
95% Confidence Interval for Mu	
347.169	350.245
95% Confidence Interval for Sigma	
12.095	14.278
95% Confidence Interval for Median	
344.932	349.758

Descriptive Statistics



Variable: Breadth

Anderson-Darling Normality Test

A-Squared:	1.115
P-Value:	0.006
Mean	150.796
StDev	5.164
Variance	26.6646
Skewness	-1.1E-01
Kurtosis	0.293729
N	281
Minimum	135.890
1st Quartile	147.828
Median	150.876
3rd Quartile	153.924
Maximum	169.926

95% Confidence Interval for Mu

150.190	151.403

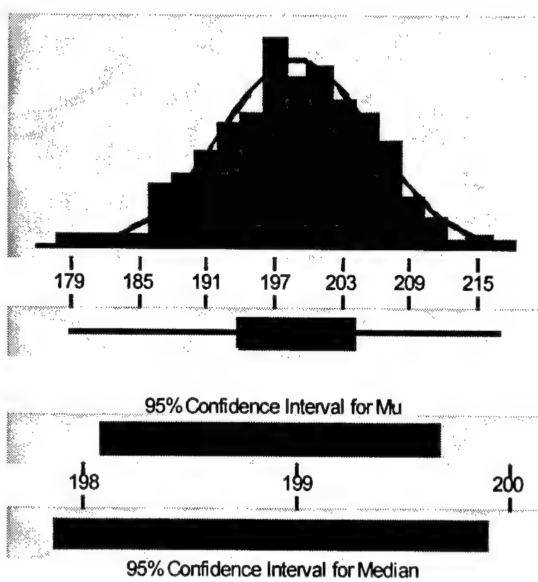
95% Confidence Interval for Sigma

4.769	5.630

95% Confidence Interval for Median

150.876	151.892

Descriptive Statistics



Variable: Length

Anderson-Darling Normality Test

A-Squared:	0.564
P-Value:	0.143
Mean	198.870
StDev	6.748
Variance	45.5346
Skewness	-1.9E-01
Kurtosis	-5.5E-02
N	281
Minimum	178.816
1st Quartile	193.802
Median	198.882
3rd Quartile	203.962
Maximum	216.916

95% Confidence Interval for Mu

198.078	199.663

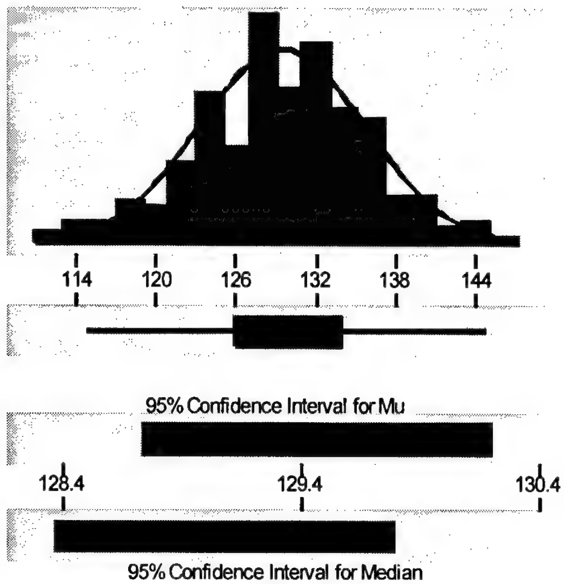
95% Confidence Interval for Sigma

6.232	7.357

95% Confidence Interval for Median

197.866	199.898

Descriptive Statistics



Variable: Height

Anderson-Darling Normality Test

A-Squared:	0.639
P-Value:	0.095
Mean	129.469
StDev	6.243
Variance	38.9748
Skewness	-5.9E-02
Kurtosis	3.35E-02
N	281
Minimum	112.776
1st Quartile	125.984
Median	128.778
3rd Quartile	133.858
Maximum	145.796

95% Confidence Interval for Mu

128.735	130.202

95% Confidence Interval for Sigma

5.766	6.807

95% Confidence Interval for Median

128.373	129.794

APPENDIX B

Output Files from Computer Program MIDPOINTS

Output File Glabmax: This file provides the distance from the midpoint to the maximum point as defined for Midpoint Method 1.

Nomenclature:

filename = filename of head scan data.
maxdis = distance from midpoint to maximum point as defined for Midpoint Method 1.
xmid, ymid, and zmid = Cartesian coordinates of the midpoint.
xmax, ymax, and zmax = Cartesian coordinates of the maximum point.

filename	maxdis	xmid	ymid	zmid	xmax	ymax	zmax
a001.xyz	212.55	8.27	230.54	-5.81	106.86	229.76	33.85
a002.xyz	214.38	1.17	252.43	1.16	100.93	251.64	40.36
a003.xyz	207.31	5.41	222.73	-2.24	97.78	223.51	44.79
a006.xyz	205.47	-9.97	247.73	0.21	86	248.52	36.87
a028.xyz	202.18	-4.19	238.36	0.25	88.99	237.58	39.45
a032.xyz	215.1	-4.86	243.04	0.56	91.83	243.83	47.65
a033.xyz	201.22	-1.48	241.48	6.1	90.47	240.7	46.94
a034.xyz	214.48	-24.56	243.04	-7.59	77.1	243.83	26.52
a039.xyz	210.65	-11.06	233.67	-1.21	88.19	232.89	34.02
a043.xyz	203.69	-12.46	261.8	-1.97	83.11	262.58	33.24
a045.xyz	199.13	7.3	235.23	3.24	98.32	234.45	43.58
a046.xyz	214.37	-4.77	244.61	1.24	97.43	245.39	33.52
a048.xyz	211.55	-22.69	210.22	-9.4	75.03	211.01	31.08
a049.xyz	214.45	-22.82	240.7	-6.68	75.23	240.7	36.72
a050.xyz	208.07	-3.45	250.86	0.2	95.35	250.08	32.8
a051.xyz	209.51	-1.2	222.73	7.87	95.53	221.95	48.08

a052.xyz	199.71	-2.58	222.73	0.8	87.44	223.51	44.01
a053.xyz	205.26	-11.78	230.54	3.95	85.5	231.32	36.65
a054.xyz	207.41	0.8	240.7	7.75	98.15	240.7	43.51
a055.xyz	202.34	-5.43	243.05	5.98	89.9	242.26	39.85
a056.xyz	206.66	12.3	236.8	6.73	110.3	236.01	39.47
a057.xyz	206.85	-3.13	238.36	2.9	93.84	237.58	38.87
a058.xyz	218.91	-7.54	238.36	-0.4	92.6	239.14	43.8
a059.xyz	207.8	4.98	282.12	3.6	99.93	281.34	45.77
a060.xyz	201.53	-11.71	216.47	1.71	81.15	217.26	40.85
a061.xyz	213.33	-6.34	239.92	-0.95	89.28	239.14	46.32
a062.xyz	205.38	-2.67	225.85	1.6	91.73	226.63	42.02
a063.xyz	211.61	7.99	262.58	7.85	109.1	262.58	39.04
a064.xyz	214.19	-27.87	233.67	-11.98	73.58	232.89	22.32
a065.xyz	212.95	7	224.29	4.31	105.32	223.51	45.15
a066.xyz	206.61	-1.24	215.69	-7.14	96.38	215.69	26.67
a068.xyz	198.6	-10.93	211.79	-1.62	83.61	211.01	28.76
a069.xyz	212.1	-30.79	246.17	-10.97	68.46	246.95	26.41
a070.xyz	203.7	-11.31	233.67	8.63	81.62	232.89	50.29
a071.xyz	209.6	9.7	271.18	9.87	107.45	270.4	47.63
a072.xyz	212.06	0.86	222.73	4.42	101.09	223.51	39
a073.xyz	203.92	13.84	239.92	3.61	108.38	239.14	41.81
a074.xyz	202.93	-9.44	241.48	1.38	83.97	240.7	40.99
a075.xyz	205.62	-32.31	225.85	2.72	63.78	226.63	39.3
a079.xyz	200.37	-16.64	218.82	-2.07	73.5	218.82	41.64
a080.xyz	210.65	3.73	238.36	-0.75	103.13	237.58	34.07
a082.xyz	199.01	3.15	243.05	11.1	95.53	242.26	48.08
a083.xyz	204.07	-14.94	218.82	-10.44	81.26	218.82	23.57
a084.xyz	204.84	2.32	238.36	3.54	98.29	239.14	39.31
a085.xyz	206.08	0.78	233.67	3.88	93.62	234.45	48.58
a086.xyz	212.83	2.9	239.92	5.32	102.12	240.7	43.78
a095.xyz	208.29	16.88	238.36	1.61	114.52	237.58	37.83
a097.xyz	211.55	-3.62	263.37	-0.1	94.12	262.58	40.35
a098.xyz	221.84	-17.24	279	10.51	85.38	278.21	52.61
a099.xyz	210.78	-10.79	269.62	-3.53	85.08	268.84	40.24
a100.xyz	211.32	-0.61	247.73	-2.37	96.74	248.52	38.69
a112.xyz	211.02	2.53	259.46	-8.05	104.04	259.46	20.69
a113.xyz	207.89	-9.38	228.98	-8.92	89.71	228.2	22.47
a114.xyz	206.48	-3.33	266.49	0.73	92.77	267.27	38.43
a115.xyz	209.74	-3.56	240.7	-5.38	95.12	240.7	30.13
a116.xyz	205.34	-3.49	264.15	0.05	92.76	264.15	35.78
a117.xyz	201.18	-9.33	275.87	-0.72	85.45	275.09	32.96
a118.xyz	213.33	-11.48	230.54	-2.38	85.61	229.76	41.78
a119.xyz	215	-0.76	261.8	-10.4	97.87	261.02	32.33
a120.xyz	203.87	-10.02	253.99	-5.65	85.66	253.21	29.47
a121.xyz	199.99	-9.27	246.17	-6.35	83.48	246.95	31.03
a122.xyz	211.81	6.49	252.43	0.53	104.61	251.64	40.35
a124.xyz	206.53	-0.86	246.95	-5.77	95.44	246.95	31.53
a125.xyz	209.35	0.23	247.73	-1.28	100.01	248.52	30.34
a126.xyz	215.11	-2.3	251.64	-7.78	99.31	251.64	27.48

a127.xyz	205.95	-10.74	283.68	-2.43	81.92	284.47	42.5
a128.xyz	204.94	-2.98	266.49	4.2	91.22	265.71	44.52
a129.xyz	207.09	3.83	254.77	0.16	99.99	254.77	38.57
a130.xyz	213.27	-6.21	253.99	-3.9	92.76	253.21	35.78
a131.xyz	211.09	-7.44	252.43	-6.17	91.61	251.64	30.26
a132.xyz	202.83	0.03	270.4	3.41	92.97	270.4	43.97
a133.xyz	204.85	-12	271.18	-3.88	83.81	270.4	32.33
a134.xyz	199.07	7.51	280.56	-2.18	99.58	281.34	35.63
a135.xyz	213.35	3.57	271.18	-1.39	102.64	270.4	38.15
a136.xyz	209.53	-18.84	268.84	-9.48	78.53	268.84	29.19
a137.xyz	208.34	-9.93	269.62	0.04	86.44	268.84	39.59
a138.xyz	201.34	10.43	273.52	-7.42	105.26	273.52	26.37
a139.xyz	216.12	6.93	253.99	-18.2	110.21	253.21	13.59
a140.xyz	204.34	-4.14	258.68	-7.28	92.95	257.89	24.5
a141.xyz	210.67	-16.17	258.68	-5.44	82.37	257.89	31.77
a142.xyz	213.12	-0.57	286.81	-2.25	100.81	287.59	30.58
a143.xyz	202.02	0.73	263.36	-2.4	92.56	264.15	39.68
a144.xyz	198.51	-2.11	236.8	-0.82	90.49	236.01	34.91
a145.xyz	200.72	-4.7	255.55	-7.44	90.27	254.77	24.98
a146.xyz	200.55	-1.22	286.81	-7.03	92.69	286.03	28.12
a147.xyz	207.91	-8.49	260.24	0.19	88.81	261.02	36.79
a149.xyz	212.42	-9.1	250.86	-11.26	93.18	251.64	17.35
a150.xyz	219.58	0.24	269.62	-4.94	103.35	268.84	32.74
a151.xyz	224.5	-12.92	258.67	-10.08	93.42	259.46	25.85
a152.xyz	195.67	4.56	244.61	-6.74	97.7	243.83	23.2
a153.xyz	209.49	-1.02	274.31	-8.62	97.06	273.52	28.15
a154.xyz	208.63	-13.63	250.86	-5.45	83.22	250.08	33.28
a155.xyz	194.64	-3.59	236.01	-7.88	86.63	236.01	28.62
a156.xyz	211.77	-2.21	268.05	-2.8	98.11	268.84	31.08
a157.xyz	210.48	4.35	282.12	6.65	101.71	282.9	46.59
a158.xyz	211.1	4.96	257.89	-5.12	104.4	257.89	30.28
a159.xyz	202.36	-14.91	268.06	-5.34	80.36	267.27	28.75
a160.xyz	212.52	-7.33	264.93	-10.81	92.89	264.15	24.48
a162.xyz	205.77	9.48	282.12	-12.11	109.77	282.9	10.81
a163.xyz	207.34	0.62	261.8	-3.84	97.78	261.02	32.3
a164.xyz	212.6	-2.16	251.64	-11.53	99.79	251.64	18.58
a165.xyz	208.89	-12.06	253.99	-5.85	85.78	253.21	30.69
a166.xyz	209.13	-10.44	282.9	-7.04	89.86	282.9	22.51
a167.xyz	202.11	-15.08	266.49	-11.83	81.07	265.71	19.25
a168.xyz	205.91	-5.26	268.06	-3.15	92.07	267.27	30.42
a169.xyz	208.9	3.92	227.42	-10.22	102.91	226.63	23.11
a170.xyz	220.23	-14.96	272.74	-12.8	90.41	271.96	19.14
a171.xyz	200.83	4.59	274.31	2.81	95.61	273.52	45.22
a172.xyz	204.93	5.93	236.01	-1.46	103.84	236.01	28.74
a173.xyz	215.18	-10.06	264.93	-10	90.76	264.15	27.53
a174.xyz	207.18	-6.76	267.27	-9.67	91.19	267.27	24.03
a175.xyz	208.41	-6.03	261.8	-1.77	90.5	261.02	37.49
a176.xyz	212.59	1.57	255.55	-7.76	102.05	254.77	26.9
a177.xyz	205.17	-8	232.11	4.31	87.62	231.32	41.44

a178.xyz	208.43	-6.4	267.27	-7.03	88.75	267.27	35.49
a179.xyz	209.94	-5.77	289.94	-2.31	91.69	289.15	36.67
a182.xyz	206.69	-7.81	264.15	-11.96	90.01	264.15	21.38
a183.xyz	215.74	-3.19	280.56	-9.95	100.09	281.34	21.19
a184.xyz	215.35	-4.41	275.09	-8.25	98.51	275.09	23.39
a185.xyz	207.41	-1.24	266.49	-9.87	96.69	267.27	24.22
a186.xyz	206.72	-8.21	260.24	-10.6	90.07	259.46	21.39
a187.xyz	211.8	-13.14	247.74	-2.98	83.8	246.95	39.64
a188.xyz	209.15	-8.57	249.3	-2.28	87.59	248.52	38.83
a189.xyz	211.52	-5.54	247.73	-5.77	92.32	248.52	34.32
a190.xyz	199.88	-1.17	266.49	-4.86	94.25	265.71	24.84
a191.xyz	207.27	0.88	236.8	-12.51	98.96	236.01	20.95
a192.xyz	204.86	-12.84	238.36	0.12	82.01	237.58	38.79
a194.xyz	212.36	0.34	266.49	2.85	101.14	267.27	36.19
a195.xyz	206.79	1.5	246.17	6.88	96.18	245.39	48.42
a196.xyz	200.56	9.23	227.42	-6.79	101.61	226.63	32.19
a197.xyz	207.81	4.95	253.99	-4.35	103.05	254.77	29.88
a198.xyz	207.5	4.43	265.71	-9.32	103.37	265.71	21.88
a200.xyz	207.66	2.2	274.3	-5.99	99.52	275.09	30.19
a201.xyz	204.47	10.35	257.11	-0.28	104.23	256.33	40.21
a202.xyz	211.31	0.08	252.43	-5.49	99.52	251.64	30.19
a203.xyz	208.95	-15.83	277.43	-9.99	81.86	276.65	27.04
a204.xyz	203.13	-0.76	252.43	-2.88	95.66	251.64	29.02
a205.xyz	206.42	7.89	285.25	-6.69	105.63	284.47	26.46
a206.xyz	196.06	3.54	252.43	-4.48	95.66	251.64	29.02
a207.xyz	205.99	-9.36	244.61	-3.35	87.62	243.83	31.35
a208.xyz	202.95	3.07	256.33	-3.5	97.99	256.33	32.37
a211.xyz	192.7	5.6	252.43	-9.5	98.21	251.64	17.04
a212.xyz	192.62	0.66	236.79	-7.98	90.67	237.58	26.29
a213.xyz	194.98	8.03	253.99	3.91	98.3	253.21	40.72
a214.xyz	209.15	-0.26	252.43	-9.76	97.18	251.64	28.18
a217.xyz	206.99	8.24	253.99	-7.95	106.26	253.21	25.24
a218.xyz	209.36	1.6	266.49	-11.19	98.44	265.71	28.55
a219.xyz	199.6	4.48	246.95	3.11	97.12	246.95	40.23
a220.xyz	208.85	-2.33	268.06	-3.47	96.8	267.27	29.36
a221.xyz	200.34	-1.01	257.11	-1.1	91.29	257.89	37.82
a222.xyz	215.03	1.5	244.61	1.99	101.79	243.83	40.71
a223.xyz	216.02	-9.37	252.42	-1.79	90.22	253.21	39.99
a224.xyz	201.57	-0.66	225.85	-4.18	94.2	225.07	29.84
a225.xyz	204.49	-6.79	244.61	-0.82	87.47	243.83	38.77
a226.xyz	202.48	-2.4	235.23	-5.15	90.09	234.45	36.03
a227.xyz	207.17	2.89	236.8	0.08	103.21	236.01	25.85
a228.xyz	210.17	-7.8	263.36	-7.03	90.58	264.15	29.92
a229.xyz	199.81	-5.89	244.61	-1.74	86.65	245.39	35.89
a230.xyz	222.67	-10.95	246.17	-12.18	96.23	246.95	17.92
a232.xyz	204.73	-7.88	232.11	-6.83	88.38	231.32	28
a233.xyz	201.4	-7.49	253.99	-7.65	83.38	253.21	35.74
a234.xyz	207.95	-0.25	251.64	-12.28	97.91	251.64	21.99
a235.xyz	202.81	-5.29	247.74	-0.85	88.41	246.95	37.9

a236.xyz	212.58	3.23	236.79	-9.27	103.07	237.58	27.17
a237.xyz	199.66	-1.54	249.3	-3.44	88.67	248.52	39.31
a238.xyz	194.91	-12.73	227.42	-0.14	75.38	226.63	41.49
a240.xyz	218.07	-8.87	243.04	-8.24	97.24	243.83	16.87
a241.xyz	207.25	3.28	233.67	-7.61	99.27	232.89	31.45
a242.xyz	206.5	8.94	243.05	-7.21	107.72	242.26	22.81
a243.xyz	198.91	-4	242.26	-12.49	86.57	242.26	28.6
a244.xyz	206.78	0.18	252.43	0.74	97.74	251.64	34.97
a245.xyz	206.08	1.81	260.24	-0.7	98.83	259.46	34
a246.xyz	200.77	-4.22	230.54	-0.52	87.57	229.76	40.11
a247.xyz	205	-1.35	258.68	-2.56	94.03	257.89	34.95
a248.xyz	202.28	-10.06	232.11	-7.13	84.39	231.32	29.03
a249.xyz	203.7	-5.64	238.36	-0.13	89.64	237.58	35.85
a250.xyz	214.13	3.79	235.23	-1.44	104.61	234.45	34.56
a251.xyz	217.12	3.29	284.47	-3.04	105.07	284.47	34.71
a252.xyz	201.34	-8.86	246.17	2.4	84.53	245.39	39.98
a253.xyz	191.14	-2.89	227.42	-4.78	87.03	226.63	27.57
a254.xyz	216.3	5.31	258.68	-0.49	103.68	257.89	44.44
a255.xyz	203.46	-13.69	272.74	-6.89	77.17	271.96	38.85
a257.xyz	203.82	-9.67	227.42	-11.45	85.95	226.63	23.79
a258.xyz	199.47	-13.71	243.05	-6.8	78.82	242.26	30.4
a259.xyz	208.57	-0.25	232.11	-4.29	95.57	231.32	36.87
a260.xyz	204.82	-6.99	246.17	-3.63	83.91	245.39	43.53
a261.xyz	199.85	-2.08	264.15	0.79	95.43	264.15	22.66
a263.xyz	214.91	-6.51	247.74	2.4	93.59	246.95	41.48
a264.xyz	201.67	-1.91	261.8	5.29	87.28	261.02	52.32
a265.xyz	205.9	-12.05	222.73	-7.32	82.22	223.51	34.06
a266.xyz	206.16	4.26	253.99	-6.01	103.13	253.21	23.16
a268.xyz	204.4	-16.81	252.43	-8.38	78.64	251.64	28.14
a269.xyz	205.09	-2.57	230.54	-2.95	93.74	229.76	32.25
a270.xyz	202.25	-6.37	243.05	-0.26	89.08	242.26	33.11
a271.xyz	209.3	-10.19	255.55	-5.01	88.3	254.77	30.38
a272.xyz	204.82	-19.49	237.58	-9.36	77.18	237.58	24.45
a273.xyz	212.93	-17.64	249.3	-6.37	74.95	248.52	46.18
a275.xyz	195.75	-20.91	243.05	-3.12	72.26	242.26	26.86
a276.xyz	201.22	-9.4	266.49	-11.52	82.46	265.71	29.51
a277.xyz	210.28	-6.01	253.21	-4	86.39	253.21	46.17
a278.xyz	202.74	-4.54	232.1	-5.82	85.63	232.89	40.5
a279.xyz	208.62	-6.41	257.11	-4.92	92.16	256.33	29.2
a280.xyz	214.91	-11.37	258.67	-0.5	90.13	259.46	34.77
a281.xyz	210.95	-4.23	263.37	-6.29	95.2	262.58	28.88
a282.xyz	221.6	-2.07	252.42	-1.48	102.02	253.21	36.5
a283.xyz	205.39	-9.11	261.8	-9.72	84	261.02	33.59
a284.xyz	208.98	-9.51	250.86	-3.35	83.88	250.08	43.52
a285.xyz	208.78	-18.11	277.43	-6.51	74.77	276.65	41.15
a286.xyz	199.41	-16.4	246.17	-11.9	78.81	245.39	17.7
a287.xyz	197.74	3.27	257.11	-2.81	97.9	256.33	25.81
a288.xyz	207.85	-4.85	258.68	-7.94	88.93	257.89	36.84
a289.xyz	204.71	8.88	258.68	-3.13	103.61	257.89	35.64

a290.xyz	200.53	-5.06	255.55	-0.65	83.11	254.77	47.08
a291.xyz	201.28	-10.06	261.8	-5.51	82.87	261.02	33.14
a292.xyz	210.12	-12.1	253.99	-8.38	83.26	253.21	35.69
a293.xyz	202.4	5.73	257.11	3.45	100.55	257.89	38.79
a294.xyz	209.47	-30.29	247.74	-17.67	58.42	246.95	38.01
a295.xyz	196.42	-21.4	247.74	-12.46	69.45	246.95	24.85
a296.xyz	205.89	-9.94	249.3	-11.28	87.5	248.52	21.92
a297.xyz	202.94	-13.91	249.3	-2.45	78.03	248.52	40.48
a298.xyz	204.02	-26.85	268.06	-11.6	60.32	267.27	41.38
a299.xyz	205.15	-15.73	274.31	-5.24	77.39	273.52	37.77
a300.xyz	207.19	-23.92	238.36	-8.35	71.22	237.58	32.62
a301.xyz	196.5	-20.42	271.18	0.12	65.67	270.4	47.46
a302.xyz	207.45	-3.56	260.24	-10.5	91.79	259.46	30.33
a305.xyz	204.97	-22.76	253.99	-8.77	67.68	253.21	39.45
a307.xyz	200.52	-17.04	272.74	-0.46	74.29	271.96	40.89
a308.xyz	207.73	-4.58	253.99	7.04	90.16	253.21	49.62
a309.xyz	196.36	-7.28	261.8	0.98	83.96	261.02	37.22
a310.xyz	208.59	-12.17	274.31	-5.3	85.33	273.52	31.72
a311.xyz	203.97	-22.36	253.99	-13.26	61.65	254.77	44.56
a312.xyz	199.59	-24.17	264.93	-4.91	64.08	264.15	41.69
a314.xyz	216.57	-26.34	252.43	-8.24	70.61	251.64	40
a315.xyz	198.93	-15.93	263.37	-10.65	77.82	262.58	22.57
a316.xyz	206.67	-30.8	244.61	-8.47	63.08	243.83	34.72
a317.xyz	195.95	-27.01	246.17	-4.87	60.43	246.95	39.32
a318.xyz	203.15	-18.08	261.8	-5.59	75.65	261.02	33.53
a319.xyz	209.81	-18.84	271.18	-1.51	79.79	270.4	34.21
a320.xyz	211.38	-6.62	258.68	-3.04	89.46	257.89	40.98
a321.xyz	208.7	-16.49	257.11	0.14	74.54	256.33	51.14
a322.xyz	193.25	-14.89	247.74	-10.43	76.74	246.95	20.23
a323.xyz	212.82	-6.52	243.05	7.09	90.32	242.26	51.17
a324.xyz	208.86	-5.62	253.99	4.48	89.4	253.21	47.79
a325.xyz	200.45	-5.05	255.55	8.52	85.05	254.77	52.41
a326.xyz	207.09	-23.31	263.37	-0.85	64.4	262.58	54.19
a327.xyz	204.52	-15.81	269.62	0.53	73.99	268.84	49.44
a328.xyz	219.52	-21.03	259.46	-6.09	81.27	259.46	33.66
a329.xyz	202.44	-11.44	241.48	9.24	76.36	240.7	59.59
a330.xyz	203.42	-5.84	255.55	-5.75	88.28	254.77	32.81
a331.xyz	207.8	-16.09	258.67	8.49	77.66	259.46	53.28
a332.xyz	204.33	-17.96	272.74	-6.13	77.62	271.96	29.94
a333.xyz	216.24	-28.95	256.33	-7.64	65.52	256.33	44.95
a334.xyz	197.93	-23.82	253.99	7.98	60.78	253.21	59.31
a335.xyz	205.89	-20.88	260.24	-2.05	73	261.02	40.18
a336.xyz	213.95	-34.67	266.49	-1.2	62.72	267.27	43.03
a337.xyz	207.9	-24.19	274.31	-2.41	70.28	273.52	40.96
a338.xyz	196.67	-7.86	214.91	3.76	85.63	214.13	34.24
a339.xyz	201.46	-13.01	271.18	-0.19	79.8	270.4	38.95
a340.xyz	193.76	-15.4	258.68	2.46	65.18	257.89	56.23
a341.xyz	207.98	-8.66	255.55	5.45	87.84	256.33	44.21
a342.xyz	191.94	-17.07	255.55	3.16	72.13	254.77	38.56

a343.xyz	204.7	1.97	252.43	0.9	95.02	251.64	43.52
a344.xyz	213.6	-16	280.56	0.19	78.06	279.78	50.78
a345.xyz	207.98	-20.66	261.8	-7	70.81	262.58	42.44
a346.xyz	196.6	-19.66	271.18	-1.91	73.2	270.4	30.32
a347.xyz	198.03	-18.97	239.92	2.86	66.88	239.14	52.19
a348.xyz	209.94	-9.36	261.8	6.17	84.92	261.02	52.33
a349.xyz	209.81	-22.03	271.18	-4.47	71.11	270.4	43.81
a350.xyz	212.75	-3.44	269.62	0.88	85.76	268.84	58.84
a351.xyz	198.55	-16.8	258.68	4	76.32	257.89	38.42
a352.xyz	199.84	-12.03	271.18	10.07	74.06	270.4	60.78
a353.xyz	207.24	-20.79	255.55	-2.04	77.1	254.77	31.94
a354.xyz	199.65	-24.32	236.8	-1.47	66.53	236.01	39.88
a355.xyz	205.17	-19.97	261.8	3.95	73.83	261.02	45.49
a356.xyz	208.78	-39.84	271.18	-5.24	54.51	270.4	39.4
a357.xyz	203.25	-25.47	274.31	5.55	63.96	273.52	53.81
a358.xyz	196.06	-33.7	258.68	-3.95	57.61	257.89	31.71
a359.xyz	204.28	-41.18	264.93	-0.37	53.88	264.15	36.97
a360.xyz	215.88	-36.43	255.55	0.32	64.47	254.77	38.65
a361.xyz	204	-32.21	286.81	-0.08	59.65	286.03	44.24
a362.xyz	212.47	-30.53	266.49	-8.18	62.14	265.71	43.76
a363.xyz	211.5	-50	268.06	1.66	44.51	267.27	49.1
a365.xyz	212.13	-31.77	262.58	-0.7	66.29	262.58	39.73

Output File Glabmidtrag: This file provides the x,y, and z Cartesian coordinates of the point between the left and right tragions used in the determination of the midpoint.

Filename	x	y	z
a001.xyz	98.26	89.55	201.18
a002.xyz	99.31	89.55	201.89
a003.xyz	96.38	90.46	209.04
a006.xyz	97.47	90.46	200.45
a028.xyz	95.23	89.53	199.41
a032.xyz	99.64	90.45	207.67
a033.xyz	92.66	89.52	204.99
a034.xyz	107.66	90.42	199.91
a039.xyz	95.42	89.53	202.53
a043.xyz	95.03	90.47	200.77
a045.xyz	87.51	89.49	205.37
a046.xyz	103.55	90.43	197.55
a048.xyz	101.99	90.44	201.91
a049.xyz	108.78	90	204.07
a050.xyz	101.37	89.56	198.64
a051.xyz	100.66	89.55	203.64
a052.xyz	92.51	90.48	207.03
a053.xyz	94.82	90.47	198.78
a054.xyz	102.23	90	200.64
a055.xyz	86.82	89.48	200.4
a056.xyz	96.87	89.54	200.12
a057.xyz	98.38	89.54	199.86
a058.xyz	101.36	90.44	203.53
a059.xyz	90.02	89.5	205.72
a060.xyz	97.76	90.46	202.36
a061.xyz	95.29	89.53	205.88
a062.xyz	93.49	90.48	203.59
a063.xyz	98.07	90	196.18
a064.xyz	105.4	89.58	202.36
a065.xyz	98.34	89.55	202.84
a066.xyz	97.83	90	198.42
a068.xyz	95.42	89.53	199.03
a069.xyz	104.14	90.43	200.06
a070.xyz	92.22	89.51	203.37
a071.xyz	92.94	89.52	201.84
a072.xyz	98.79	90.45	200.47
a073.xyz	97	89.54	203.21
a074.xyz	91.4	89.51	203.05
a075.xyz	95.75	90.47	199.34
a079.xyz	94.35	90	206.67
a080.xyz	94.58	89.53	201.27
a082.xyz	91.99	89.51	200.6
a083.xyz	108.31	90	200.37
a084.xyz	101.69	90.44	200.05

a085.xyz	99.65	90.45	205.84
a086.xyz	104.87	90.43	200.44
a095.xyz	92.52	89.52	202.69
a097.xyz	92.21	89.51	204.04
a098.xyz	96.5	89.54	203.46
a099.xyz	91.78	89.51	205.77
a100.xyz	100.88	90.45	205.45
a112.xyz	96.33	90	197.45
a113.xyz	93.53	89.52	196.51
a114.xyz	96.78	90.46	199.16
a115.xyz	97.91	90	200.28
a116.xyz	93.79	90	201.15
a117.xyz	86.6	89.48	200.27
a118.xyz	100	89.55	204.77
a119.xyz	102.3	89.56	201.48
a120.xyz	92.11	89.51	199.25
a121.xyz	93.53	90.48	200.86
a122.xyz	94.66	89.53	203.1
a124.xyz	95.66	90	201.03
a125.xyz	98.83	90.45	197.1
a126.xyz	103.62	90	196.06
a127.xyz	97.92	90.46	205.71
a128.xyz	90.86	89.51	200.93
a129.xyz	90.82	90	202.58
a130.xyz	99.66	89.55	200.26
a131.xyz	93.94	89.52	198.82
a132.xyz	94.1	90	200.96
a133.xyz	83.93	89.47	200.13
a134.xyz	89.42	90.5	205.13
a135.xyz	98.42	89.55	201.39
a136.xyz	99.59	90	200.63
a137.xyz	89.9	89.5	201.64
a138.xyz	90.34	90	200.66
a139.xyz	101.32	89.56	198.28
a140.xyz	89.33	89.5	200.62
a141.xyz	102.21	89.56	202.3
a142.xyz	93.41	90.48	197.09
a143.xyz	96.54	90.47	204.39
a144.xyz	91.78	89.51	199.46
a145.xyz	90.96	89.51	199.54
a146.xyz	89.23	89.5	201.54
a147.xyz	91.84	90.49	201.18
a149.xyz	100.98	90.44	192.09
a150.xyz	94.28	89.52	200.71
a151.xyz	107.48	90.42	198.18
a152.xyz	86.99	89.49	197.03
a153.xyz	90.59	89.51	201.52
a154.xyz	93.08	89.52	199.69
a155.xyz	91.02	90	200.46

a156.xyz	93.12	90.48	198.72
a157.xyz	88.32	90.51	200.59
a158.xyz	92.39	90	200.06
a159.xyz	91.41	89.51	198.06
a160.xyz	94.56	89.53	199.63
a162.xyz	95.64	90.47	195.7
a163.xyz	93.32	89.52	200.62
a164.xyz	96.33	90	195.46
a165.xyz	93.09	89.52	199.21
a166.xyz	93.84	90	194.72
a167.xyz	91.41	89.51	196.36
a168.xyz	87.63	89.49	200.34
a169.xyz	97.17	89.54	199.03
a170.xyz	103.47	89.57	199.2
a171.xyz	84.77	89.47	205.04
a172.xyz	91.51	90	197.65
a173.xyz	96.44	89.54	198.78
a174.xyz	89.86	90	199.71
a175.xyz	90.39	89.51	202.03
a176.xyz	102.98	89.57	200.32
a177.xyz	92.9	89.52	198.12
a178.xyz	96.16	90	204.54
a179.xyz	95.15	89.53	200.69
a182.xyz	96.37	90	199.74
a183.xyz	103.13	90.43	195.34
a184.xyz	104.47	90	199.97
a185.xyz	97.95	90.46	198.41
a186.xyz	99.38	89.55	197.76
a187.xyz	98.56	89.54	203.4
a188.xyz	98.46	89.55	200.45
a189.xyz	93.22	90.48	201.04
a190.xyz	85.46	89.48	200.36
a191.xyz	98.52	89.54	200.7
a192.xyz	87.06	89.49	201.86
a194.xyz	101.32	90.44	198.52
a195.xyz	93.99	89.52	204.24
a196.xyz	89.82	89.5	205.18
a197.xyz	99.83	90.45	200.58
a198.xyz	96.6	90	199.35
a200.xyz	97.14	90.46	198.08
a201.xyz	96.49	89.54	202.18
a202.xyz	95.47	89.53	198.6
a203.xyz	92.76	89.52	200.53
a204.xyz	94.44	89.52	197.19
a205.xyz	92.9	89.52	198.65
a206.xyz	92.7	89.52	198.8
a207.xyz	90.01	89.5	200.91
a208.xyz	98.19	90	198.79
a211.xyz	87.15	89.48	198.86

a212.xyz	85.46	90.52	200.64
a213.xyz	86.98	89.48	203.64
a214.xyz	93.69	89.52	201.07
a217.xyz	99.06	89.55	200.36
a218.xyz	88.69	89.5	204.33
a219.xyz	90.75	90	199.13
a220.xyz	96.3	89.53	197.29
a221.xyz	92.36	90.49	202.51
a222.xyz	99.64	89.55	201
a223.xyz	96.91	90.46	202.55
a224.xyz	92.88	89.52	201.38
a225.xyz	95.3	89.53	202.88
a226.xyz	90.53	89.51	202.01
a227.xyz	95.2	89.53	195.89
a228.xyz	96.29	90.47	200.34
a229.xyz	92.71	90.48	204.17
a230.xyz	102.66	90.44	197.76
a232.xyz	91.19	89.51	199.19
a233.xyz	84.02	89.47	204.07
a234.xyz	96.17	90	201.94
a235.xyz	95.54	89.53	201.78
a236.xyz	102.36	90.44	200.36
a237.xyz	85.69	89.48	203.98
a238.xyz	84.75	89.47	205.02
a240.xyz	96.92	90.46	193.04
a241.xyz	94.46	89.53	202.19
a242.xyz	96.45	89.54	199.3
a243.xyz	96.49	90	204.47
a244.xyz	93.61	89.52	200.51
a245.xyz	94.55	89.53	200.23
a246.xyz	89.93	89.5	205.47
a247.xyz	89.59	89.5	201.95
a248.xyz	96.06	89.53	200.52
a249.xyz	90.39	89.5	199.52
a250.xyz	101.11	89.56	200.79
a251.xyz	98.03	90	198.73
a252.xyz	92.81	89.52	202.07
a253.xyz	87.63	89.49	202.78
a254.xyz	94.57	89.53	204.03
a255.xyz	85.02	89.48	206.5
a257.xyz	92.59	89.52	200.07
a258.xyz	91.5	89.51	202.31
a259.xyz	93.26	89.52	204.53
a260.xyz	95.6	89.53	206.92
a261.xyz	88.31	90	191
a263.xyz	95.12	89.53	200.58
a264.xyz	90.91	89.51	208.46
a265.xyz	92.89	90.48	202.61
a266.xyz	93.05	89.52	196.29

a268.xyz	84.73	89.47	198.85
a269.xyz	94.67	89.53	200.16
a270.xyz	92.74	89.52	199.5
a271.xyz	92.68	89.52	197.41
a272.xyz	97.42	90	200.85
a273.xyz	93.9	89.52	209.16
a275.xyz	88.17	89.49	198.65
a276.xyz	90.68	89.51	201.66
a277.xyz	97.3	90	204.63
a278.xyz	97.13	90.46	207.26
a279.xyz	97.12	89.54	201.01
a280.xyz	99.8	90.45	199.81
a281.xyz	89.18	89.5	200.28
a282.xyz	102.75	90.44	198.86
a283.xyz	93.49	89.52	203.47
a284.xyz	96.51	89.54	208.61
a285.xyz	95.69	89.53	206.32
a286.xyz	84.8	89.47	196.63
a287.xyz	83.3	89.46	195.49
a288.xyz	97.41	89.54	204.81
a289.xyz	99.29	89.55	202.95
a290.xyz	93.68	89.52	209.47
a291.xyz	88.99	89.5	200.13
a292.xyz	91.29	89.51	205.93
a293.xyz	96.38	90.47	200.99
a294.xyz	102.46	89.56	212.79
a295.xyz	88.43	89.49	202.39
a296.xyz	95.25	89.53	199.29
a297.xyz	91	89.51	202.4
a298.xyz	93.61	89.52	211.54
a299.xyz	94.33	89.52	205.5
a300.xyz	101.63	89.56	203.63
a301.xyz	82.8	89.46	208.81
a302.xyz	97.75	89.54	203.17
a305.xyz	94.44	89.53	207.06
a307.xyz	82.85	89.46	203.88
a308.xyz	90.11	89.5	202.17
a309.xyz	90.81	89.51	200.12
a310.xyz	94.82	89.53	200.16
a311.xyz	95.07	90.47	215.83
a312.xyz	96.38	89.54	208.58
a314.xyz	100.34	89.55	206.24
a315.xyz	91.88	89.51	200.3
a316.xyz	96.85	89.54	205.47
a317.xyz	83.07	90.54	206.18
a318.xyz	90.21	89.5	201.61
a319.xyz	94.7	89.53	198.36
a320.xyz	90.79	89.51	202.34
a321.xyz	91.19	89.51	209.08

a322.xyz	79.1	89.43	198.37
a323.xyz	97.22	89.54	206.61
a324.xyz	97.09	89.54	204.74
a325.xyz	90.17	89.5	203.37
a326.xyz	96.07	89.53	210.65
a327.xyz	92.24	89.51	208.2
a328.xyz	103.28	90	203.24
a329.xyz	96.82	89.54	210.37
a330.xyz	89.56	89.5	202.2
a331.xyz	97.58	90.46	206.3
a332.xyz	89.52	89.5	200.81
a333.xyz	100.52	90	210.42
a334.xyz	91.02	89.51	208.79
a335.xyz	96.99	90.46	204.15
a336.xyz	105.38	90.42	204.47
a337.xyz	95.46	89.53	202.88
a338.xyz	88.38	89.49	198.06
a339.xyz	91.91	89.51	201.48
a340.xyz	92.4	89.51	216.01
a341.xyz	94.99	90.47	200.74
a342.xyz	88.51	89.49	201.14
a343.xyz	101.53	89.56	205.46
a344.xyz	95.26	89.53	206.19
a345.xyz	98.43	90.46	210.54
a346.xyz	84.22	89.47	198.92
a347.xyz	94.13	89.52	209.89
a348.xyz	94.23	89.53	205.28
a349.xyz	95.05	89.53	208.34
a350.xyz	96	89.53	213.07
a351.xyz	85.73	89.48	199.85
a352.xyz	88.58	89.5	209.26
a353.xyz	91.64	89.51	199.45
a354.xyz	85.16	89.47	203.99
a355.xyz	93.31	89.52	204.97
a356.xyz	93.34	89.52	205.49
a357.xyz	86.85	89.48	211.71
a358.xyz	87.86	89.49	200.62
a359.xyz	92.46	89.52	201.53
a360.xyz	97.01	89.54	199.74
a361.xyz	84.75	89.47	203.44
a362.xyz	101.05	89.56	209.23
a363.xyz	94.54	89.53	206.23
a365.xyz	89.51	90	203.74

Output file Glabnuch: This file provides the distance and the direction angles between the nuchales and the glabella (heading n-g), the distance between the maximum point, defined in Midpoint Method 1 and the glabella (heading maxdis), and distance and direction angles between the glabella and the midpoint defined in Midpoint Method 1 (heading middis).

filename	n-g	phi	theta	maxdis	middis	phi	theta
a001.xyz	206.77	75.11	201.49	212.55	106.28	89.58	201.91
a002.xyz	204.98	75.88	201.76	214.38	107.19	89.58	201.45
a003.xyz	200.25	77.37	206.78	207.31	103.66	90.43	206.98
a006.xyz	200.77	70.92	201	205.47	102.74	90.44	200.91
a028.xyz	199.96	75.51	203.09	202.18	101.09	89.56	202.81
a032.xyz	206.79	76.45	206.17	215.1	107.55	90.42	205.96
a033.xyz	195.39	79.87	203.45	201.22	100.61	89.56	203.95
a034.xyz	210.99	80.62	199.05	214.48	107.24	90.42	198.55
a039.xyz	201.97	72.9	199.96	210.65	105.32	89.57	199.55
a043.xyz	198.05	69.68	200.34	203.69	101.84	90.44	200.22
a045.xyz	191.17	72.88	204.26	199.13	99.57	89.55	203.91
a046.xyz	208.4	72.09	198.02	214.37	107.19	90.42	197.53
a048.xyz	204.98	83.43	202.75	211.55	105.78	90.42	202.5
a049.xyz	206.66	79.98	203.51	214.45	107.22	90	203.87
a050.xyz	207.91	72.05	198.23	208.07	104.04	89.57	198.26
a051.xyz	201.4	76.54	202.63	209.51	104.75	89.57	202.58
a052.xyz	194.73	79.83	205.93	199.71	99.85	90.45	205.65
a053.xyz	192.78	73.03	198.61	205.26	102.63	90.44	198.58
a054.xyz	199.22	75.46	200.57	207.41	103.7	90	200.17
a055.xyz	197.13	72.46	199.86	202.34	101.17	89.56	199.56
a056.xyz	199.93	72.25	197.99	206.66	103.33	89.57	198.47
a057.xyz	201.58	70.52	200.16	206.85	103.42	89.57	200.35
a058.xyz	206.36	77.75	203.39	218.91	109.46	90.41	203.82
a059.xyz	204.33	67.99	203.88	207.8	103.9	89.57	203.95
a060.xyz	191.19	78.21	202.56	201.53	100.77	90.45	202.85
a061.xyz	204.95	71.32	206.5	213.33	106.66	89.58	206.31
a062.xyz	197.2	77.18	203.4	205.38	102.69	90.44	203.18
a063.xyz	205.32	71.81	197.28	211.61	105.8	90	197.14
a064.xyz	209.07	77.48	199	214.19	107.1	89.58	198.68
a065.xyz	209.19	75.73	202.16	212.95	106.47	89.58	202.56
a066.xyz	194.2	71.71	199.54	206.61	103.3	90	199.1
a068.xyz	194.43	78.88	198.07	198.6	99.3	89.55	197.82
a069.xyz	204.07	78.07	200.18	212.1	106.05	90.42	200.64
a070.xyz	193.03	69.62	204.06	203.7	101.85	89.56	204.15
a071.xyz	208.9	77.47	201.05	209.6	104.8	89.57	201.12
a072.xyz	204.89	76.32	198.56	212.06	106.03	90.42	199.03
a073.xyz	201.08	75.14	202.43	203.92	101.96	89.56	202
a074.xyz	198.38	65.82	203.44	202.93	101.46	89.56	202.98
a075.xyz	196.03	78.04	200.74	205.62	102.81	90.44	200.84

a079.xyz	194.43	73.66	206.13	200.37	100.19	90	205.87
a080.xyz	202.84	71.58	199.73	210.65	105.32	89.57	199.31
a082.xyz	189.8	72.26	202.02	199.01	99.5	89.55	201.82
a083.xyz	197.89	81.83	199.06	204.07	102.03	90	199.47
a084.xyz	192.72	77.35	200.32	204.84	102.42	90.44	200.44
a085.xyz	200.49	77.39	205.96	206.08	103.04	90.43	205.71
a086.xyz	201.92	73.82	201.33	212.83	106.41	90.42	201.19
a095.xyz	204.38	69.4	200.54	208.29	104.15	89.57	200.36
a097.xyz	202.65	68.27	202.69	211.55	105.77	89.58	202.48
a098.xyz	214.33	69.96	202.78	221.84	110.92	89.6	202.31
a099.xyz	206.78	65.91	204.98	210.78	105.39	89.57	204.54
a100.xyz	202.68	71.57	202.66	211.32	105.66	90.42	202.87
a112.xyz	205.39	71.82	196.18	211.02	105.51	90	195.81
a113.xyz	206	75.05	197.74	207.89	103.95	89.57	197.58
a114.xyz	201.88	73.82	201.03	206.48	103.24	90.43	201.42
a115.xyz	202.83	69.24	200.03	209.74	104.87	90	199.79
a116.xyz	198.72	73.55	200.62	205.34	102.67	90	200.37
a117.xyz	193.4	70.16	199.4	201.18	100.59	89.56	199.56
a118.xyz	205.15	76.79	204.93	213.33	106.67	89.58	204.46
a119.xyz	211.43	70.57	203.52	215	107.5	89.58	203.43
a120.xyz	199.09	72.17	199.96	203.87	101.93	89.56	200.16
a121.xyz	188.44	78.03	201.52	199.99	100	90.45	201.95
a122.xyz	203.07	71.6	202.19	211.81	105.9	89.58	202.09
a124.xyz	196.7	81.78	201.35	206.53	103.27	90	201.17
a125.xyz	203.65	77.59	197.95	209.35	104.67	90.43	197.58
a126.xyz	206.99	76.46	199.08	215.11	107.55	90	199.14
a127.xyz	200.26	76	205.49	205.95	102.98	90.44	205.87
a128.xyz	202.33	75.23	203.04	204.94	102.47	89.56	203.18
a129.xyz	202.91	70.19	201.5	207.09	103.55	90	201.77
a130.xyz	206.68	78.66	201.58	213.27	106.64	89.58	201.85
a131.xyz	203.12	70.68	200.11	211.09	105.55	89.57	200.19
a132.xyz	194.64	78.89	203.81	202.83	101.41	90	203.58
a133.xyz	194.01	69.73	200.91	204.85	102.42	89.56	200.7
a134.xyz	196.52	63.55	202.68	199.07	99.54	90.45	202.33
a135.xyz	211.64	78.07	201.45	213.35	106.68	89.58	201.76
a136.xyz	201.69	75.64	201.25	209.53	104.76	90	201.66
a137.xyz	202.25	71.53	202.7	208.34	104.17	89.57	202.32
a138.xyz	198.18	69.7	200.07	201.34	100.67	90	199.61
a139.xyz	205.2	75.44	196.65	216.12	108.06	89.59	197.11
a140.xyz	193.63	74.07	198.45	204.34	102.17	89.56	198.12
a141.xyz	208.84	73.47	200.39	210.67	105.34	89.57	200.69
a142.xyz	201.36	74.23	197.74	213.12	106.56	90.42	197.94
a143.xyz	194.33	76.04	204.72	202.02	101.01	90.44	204.62
a144.xyz	191.54	77.75	201.1	198.51	99.26	89.55	201.1
a145.xyz	197.67	69.64	199.13	200.72	100.36	89.55	198.85
a146.xyz	195.8	71.86	200.8	200.55	100.28	89.55	200.52
a147.xyz	204.27	73.09	201.09	207.91	103.96	90.43	200.61
a149.xyz	203.25	77.57	195.69	212.42	106.21	90.42	195.62
a150.xyz	211.36	71.46	200.31	219.58	109.79	89.59	200.07

a151.xyz	216.97	79.21	199.12	224.5	112.25	90.4	198.67
a152.xyz	187.22	74.01	197.88	195.67	97.83	89.54	197.82
a153.xyz	209.41	67.16	200.61	209.49	104.75	89.57	200.55
a154.xyz	203.44	69.3	201.54	208.63	104.31	89.57	201.8
a155.xyz	188.07	72.59	202.04	194.64	97.32	90	202.02
a156.xyz	209.64	73.54	198.72	211.77	105.88	90.42	198.66
a157.xyz	203.88	64.58	202.63	210.48	105.24	90.43	202.3
a158.xyz	208.49	77.45	200.07	211.1	105.55	90	199.59
a159.xyz	197.84	71.58	199.69	202.36	101.18	89.56	199.69
a160.xyz	207.64	74.72	199.68	212.52	106.26	89.58	199.4
a162.xyz	202.29	71.06	193.07	205.77	102.89	90.44	192.88
a163.xyz	198.99	71.21	200.31	207.34	103.67	89.57	200.41
a164.xyz	205.06	73.62	196.93	212.6	106.3	90	196.45
a165.xyz	204.21	71.25	200.55	208.89	104.44	89.57	200.48
a166.xyz	204.68	69.43	196.35	209.13	104.56	90	196.42
a167.xyz	195.99	71.88	198.3	202.11	101.05	89.56	197.91
a168.xyz	204.52	68.95	199.39	205.91	102.96	89.56	199.03
a169.xyz	199.66	75.49	198.43	208.9	104.45	89.57	198.61
a170.xyz	213.12	75.56	196.92	220.23	110.11	89.59	196.86
a171.xyz	205.62	64.81	205.3	200.83	100.41	89.55	204.98
a172.xyz	200.1	68.46	197.31	204.93	102.47	90	197.14
a173.xyz	208.79	72.58	200.15	215.18	107.59	89.58	200.42
a174.xyz	197.17	66.15	198.82	207.18	103.59	90	198.99
a175.xyz	199.24	65.93	201.8	208.41	104.21	89.57	202.13
a176.xyz	208.94	75.27	199.25	212.59	106.3	89.58	199.03
a177.xyz	195.57	70.39	201.63	205.17	102.59	89.56	201.22
a178.xyz	198.65	75.42	204.57	208.43	104.22	90	204.08
a179.xyz	209.41	69	201.5	209.94	104.97	89.57	201.8
a182.xyz	199.93	78.73	199.11	206.69	103.34	90	198.82
a183.xyz	210.65	77.14	196.7	215.74	107.87	90.41	196.78
a184.xyz	210.68	76.7	196.65	215.35	107.67	90	197.09
a185.xyz	198.87	73.09	199.66	207.41	103.7	90.43	199.2
a186.xyz	195.64	73.28	198.51	206.72	103.36	89.57	198.03
a187.xyz	207.74	71.12	204.16	211.8	105.9	89.58	203.73
a188.xyz	207.68	69.29	203.09	209.15	104.57	89.57	203.15
a189.xyz	204.13	70.31	202.75	211.52	105.76	90.42	202.27
a190.xyz	196.54	66.57	197.26	199.88	99.94	89.55	197.29
a191.xyz	201.58	74.25	198.99	207.27	103.63	89.57	198.84
a192.xyz	196.35	68.52	202.06	204.86	102.43	89.56	202.18
a194.xyz	207.93	70.23	198.51	212.36	106.18	90.42	198.3
a195.xyz	202.62	74.34	203.5	206.79	103.4	89.57	203.68
a196.xyz	192.4	70.05	203.12	200.56	100.28	89.55	202.88
a197.xyz	201.08	72.35	199.5	207.81	103.91	90.43	199.24
a198.xyz	202.13	74.76	197.52	207.5	103.75	90	197.5
a200.xyz	197.22	76.25	200.69	207.66	103.83	90.43	200.39
a201.xyz	198.36	71.63	203.52	204.47	102.23	89.56	203.33
a202.xyz	207.61	68.81	199.37	211.31	105.66	89.58	199.74
a203.xyz	201.17	68.58	201.05	208.95	104.48	89.57	200.76
a204.xyz	199.26	75.46	198.38	203.13	101.56	89.56	198.31

a205.xyz	201	67.12	198.93	206.42	103.21	89.57	198.73
a206.xyz	192.33	68.55	199.62	196.06	98.03	89.54	199.98
a207.xyz	199.52	68.88	199.41	205.99	103	89.57	199.69
a208.xyz	194.91	67.86	200.38	202.95	101.47	90	200.7
a211.xyz	187.6	69.52	196.25	192.7	96.35	89.53	195.99
a212.xyz	190.42	73.31	200.79	192.62	96.31	90.47	200.85
a213.xyz	194	73.62	202.53	194.98	97.49	89.54	202.18
a214.xyz	199.76	74.1	201.73	209.15	104.57	89.57	201.27
a217.xyz	198.73	70.23	198.26	206.99	103.49	89.57	198.7
a218.xyz	209.04	68.51	202.46	209.36	104.68	89.57	202.31
a219.xyz	195.18	71.32	202.13	199.6	99.8	90	201.84
a220.xyz	201.66	72.87	198.14	208.85	104.42	89.57	198.33
a221.xyz	193.95	72.65	202.89	200.34	100.17	90.45	202.86
a222.xyz	204.23	73.09	201.04	215.03	107.51	89.58	201.11
a223.xyz	208.69	73.91	202.96	216.02	108.01	90.42	202.76
a224.xyz	197.93	72.06	199.89	201.57	100.79	89.56	199.73
a225.xyz	198.86	70.25	202.68	204.49	102.24	89.56	202.79
a226.xyz	195.56	65.44	203.73	202.48	101.24	89.56	204
a227.xyz	200.12	74.13	194.77	207.17	103.59	89.57	194.41
a228.xyz	206.13	69.12	201.08	210.17	105.09	90.43	200.59
a229.xyz	189.53	73.22	202.38	199.81	99.91	90.45	202.13
a230.xyz	220.54	69.68	196.08	222.67	111.34	90.4	195.69
a232.xyz	197.41	71.54	199.54	204.73	102.37	89.56	199.89
a233.xyz	195.67	70.88	205.73	201.4	100.7	89.56	205.52
a234.xyz	206.95	70.13	199.7	207.95	103.98	90	199.24
a235.xyz	199.14	72.17	202.72	202.81	101.4	89.56	202.46
a236.xyz	204.55	78.54	199.7	212.58	106.29	90.42	200.05
a237.xyz	192.21	73.95	205.78	199.66	99.83	89.55	205.35
a238.xyz	191.92	69.5	205	194.91	97.46	89.54	205.29
a240.xyz	209.1	62.39	192.92	218.07	109.04	90.41	193.32
a241.xyz	200.12	71.32	201.92	207.25	103.63	89.57	202.14
a242.xyz	200.31	70.87	197.35	206.5	103.25	89.57	196.9
a243.xyz	190.6	77.21	203.94	198.91	99.45	90	204.4
a244.xyz	200.02	69.41	199.3	206.78	103.39	89.57	199.34
a245.xyz	202.43	69.2	199.74	206.08	103.04	89.57	199.68
a246.xyz	191.79	77.29	204.13	200.77	100.39	89.56	203.88
a247.xyz	201.87	71.49	201.86	205	102.5	89.56	201.47
a248.xyz	200.12	77.83	201.31	202.28	101.14	89.56	200.95
a249.xyz	212.2	67.94	200.65	203.7	101.85	89.56	200.69
a250.xyz	204.71	73.14	200.11	214.13	107.06	89.58	199.65
a251.xyz	213.61	70.77	200.84	217.12	108.56	90	200.35
a252.xyz	200.13	68.47	202.24	201.34	100.67	89.56	201.92
a253.xyz	187.49	76.01	200.21	191.14	95.57	89.53	199.79
a254.xyz	207.08	68.76	204.72	216.3	108.15	89.59	204.55
a255.xyz	205.69	68.14	206.72	203.46	101.73	89.56	206.72
a257.xyz	197.52	76.27	200.52	203.82	101.91	89.56	200.23
a258.xyz	196.65	74.79	201.67	199.47	99.73	89.55	201.91
a259.xyz	203.98	73.07	203.71	208.57	104.29	89.57	203.24
a260.xyz	203.74	72.59	207.73	204.82	102.41	89.56	207.42

a261.xyz	192.8	72.06	192.89	199.85	99.93	90	192.65
a263.xyz	208.16	68.87	201.35	214.91	107.46	89.58	201.33
a264.xyz	199.02	67.85	208.15	201.67	100.84	89.56	207.8
a265.xyz	192.51	77.33	203.28	205.9	102.95	90.43	203.7
a266.xyz	200.02	68.45	196.84	206.16	103.08	89.57	196.44
a268.xyz	202.36	64.86	200.6	204.4	102.2	89.56	200.94
a269.xyz	203.87	73.06	200.45	205.09	102.55	89.56	200.08
a270.xyz	199.21	67.87	199.46	202.25	101.12	89.56	199.27
a271.xyz	210.12	68.62	200.11	209.3	104.65	89.57	199.77
a272.xyz	200.31	77.38	199.62	204.82	102.41	90	199.28
a273.xyz	207.34	66.45	209.81	212.93	106.47	89.58	209.58
a275.xyz	196.57	70.01	198.18	195.75	97.87	89.54	197.83
a276.xyz	197.31	72.48	204.37	201.22	100.61	89.56	204.06
a277.xyz	202.53	72.95	208.24	210.28	105.14	90	208.5
a278.xyz	193.79	78.37	207.62	202.74	101.37	90.44	207.19
a279.xyz	203.16	73.46	199.5	208.62	104.31	89.57	199.09
a280.xyz	212.14	73.74	199.64	214.91	107.46	90.42	199.16
a281.xyz	205.11	66.18	199.8	210.95	105.48	89.57	199.48
a282.xyz	211.58	77.2	200.39	221.6	110.8	90.4	200.05
a283.xyz	201.73	70.54	204.64	205.39	102.69	89.57	204.95
a284.xyz	202.67	71.57	206.87	208.98	104.49	89.57	206.65
a285.xyz	213.13	70.29	207.64	208.78	104.39	89.57	207.17
a286.xyz	196.42	66.06	197.54	199.41	99.7	89.55	197.27
a287.xyz	190.48	67.83	197.07	197.74	98.87	89.55	196.83
a288.xyz	202.82	73.89	205.73	207.85	103.92	89.57	205.52
a289.xyz	209.06	70.34	202.75	204.71	102.35	89.56	202.26
a290.xyz	199.15	66.9	208.03	200.53	100.26	89.55	208.43
a291.xyz	195.58	70.39	202.38	201.28	100.64	89.56	202.58
a292.xyz	201.66	71	205.23	210.12	105.06	89.57	204.81
a293.xyz	197.28	73.9	200.74	202.4	101.2	90.44	200.44
a294.xyz	204.5	77.19	212.06	209.47	104.74	89.57	212.11
a295.xyz	194.84	65.85	202.82	196.42	98.21	89.54	202.33
a296.xyz	195.37	73.26	199.02	205.89	102.95	89.57	198.81
a297.xyz	201.85	68.66	204.84	202.94	101.47	89.56	205.03
a298.xyz	203.21	75.3	211.65	204.02	102.01	89.56	211.3
a299.xyz	204.17	70.32	205.25	205.15	102.58	89.56	204.79
a300.xyz	201.01	76.97	202.98	207.19	103.6	89.57	203.3
a301.xyz	200.17	61.56	209.21	196.5	98.25	89.54	208.81
a302.xyz	199.51	76.87	203.21	207.45	103.73	89.57	203.18
a305.xyz	201.52	67.66	208.3	204.97	102.49	89.56	208.06
a307.xyz	194.01	71.2	204.73	200.52	100.26	89.55	204.36
a308.xyz	205.84	69.09	204.47	207.73	103.87	89.57	204.2
a309.xyz	188.82	72.67	201.17	196.36	98.18	89.55	201.66
a310.xyz	202.72	70.64	200.55	208.59	104.3	89.57	200.79
a311.xyz	194.76	76.54	214.44	203.97	101.99	90.44	214.54
a312.xyz	200.6	70.43	207.54	199.59	99.79	89.55	207.83
a314.xyz	208.41	68.9	206.74	216.57	108.28	89.59	206.45
a315.xyz	197.63	69.63	199.24	198.93	99.46	89.55	199.51
a316.xyz	198.34	72.58	205.14	206.67	103.33	89.57	204.7

a317.xyz	192.44	70.55	206.53	195.95	97.98	90.46	206.81
a318.xyz	198.76	75.43	202.59	203.15	101.57	89.56	202.66
a319.xyz	207.86	66.98	200.34	209.81	104.91	89.57	199.9
a320.xyz	202.6	65.86	204.61	211.38	105.69	89.58	204.61
a321.xyz	194.86	69.33	209.75	208.7	104.35	89.57	209.26
a322.xyz	190.84	71.37	198.76	193.25	96.63	89.54	198.5
a323.xyz	211.14	70.99	204.78	212.82	106.41	89.58	204.47
a324.xyz	201.22	72.36	204.8	208.86	104.43	89.57	204.5
a325.xyz	197.86	68.69	206.12	200.45	100.23	89.55	205.97
a326.xyz	204.33	74.47	211.69	207.09	103.54	89.57	212.11
a327.xyz	201.83	65.28	209.02	204.52	102.26	89.56	208.58
a328.xyz	214.91	73.09	200.9	219.52	109.76	90	201.24
a329.xyz	198.48	71.64	209.53	202.44	101.22	89.56	209.83
a330.xyz	203.07	69.26	202.71	203.42	101.71	89.56	202.28
a331.xyz	203.67	72.12	205.77	207.8	103.9	90.43	205.53
a332.xyz	198.26	65.8	200.39	204.33	102.17	89.56	200.68
a333.xyz	205.86	76.83	209.59	216.24	108.12	90	209.11
a334.xyz	193.78	72.15	210.84	197.93	98.96	89.55	211.25
a335.xyz	200.64	71.84	204.1	205.89	102.94	90.43	204.22
a336.xyz	210.18	72.7	204.2	213.95	106.97	90.42	204.43
a337.xyz	203.47	65.01	204.17	207.9	103.95	89.57	204.66
a338.xyz	189.08	65.59	198.36	196.67	98.34	89.55	198.06
a339.xyz	200.62	68.52	203.18	201.46	100.73	89.56	202.87
a340.xyz	188.93	73.66	213.8	193.76	96.88	89.54	213.71
a341.xyz	200.19	75.07	201.52	207.98	103.99	90.43	201.89
a342.xyz	195.86	67.97	202.11	191.94	95.97	89.53	201.64
a343.xyz	201.38	75.16	204.61	204.7	102.35	89.56	204.61
a344.xyz	207.74	76.51	208.57	213.6	106.8	89.58	208.28
a345.xyz	198.85	74.5	208.66	207.98	103.99	90.43	208.39
a346.xyz	192.63	63.5	199.57	196.6	98.3	89.55	199.14
a347.xyz	193.64	69.2	210.19	198.03	99.02	89.55	209.88
a348.xyz	207.38	68.33	206.19	209.94	104.97	89.57	206.08
a349.xyz	207.58	64.58	207.82	209.81	104.91	89.57	207.4
a350.xyz	216.92	67.99	212.93	212.75	106.37	89.58	213.02
a351.xyz	195.6	66.95	199.88	198.55	99.27	89.55	200.29
a352.xyz	196.01	70.43	210.71	199.84	99.92	89.55	210.5
a353.xyz	203.76	68.87	199.57	207.24	103.62	89.57	199.14
a354.xyz	193.53	73.58	204.71	199.65	99.83	89.55	204.47
a355.xyz	202.18	78.86	204.2	205.17	102.59	89.56	203.89
a356.xyz	203.71	73.05	205.35	208.78	104.39	89.57	205.32
a357.xyz	199.82	68.91	208.32	203.25	101.63	89.56	208.36
a358.xyz	194.08	76.97	201.5	196.06	98.03	89.54	201.33
a359.xyz	191.25	70.42	201.18	204.28	102.14	89.56	201.44
a360.xyz	208.06	74.76	200.45	215.88	107.94	89.59	200.8
a361.xyz	201.99	70.57	206.13	204	102	89.56	205.76
a362.xyz	213.28	67.14	209.01	212.47	106.23	89.58	209.27
a363.xyz	204.34	72.18	207.12	211.5	105.75	89.58	206.66
a365.xyz	208.57	73.46	202.82	212.13	106.07	90	202.41

Ouput file Ptsglabnuch: This file provides the Cartesian coordinates of the glabella and nuchales of the data from the 1990 U.S. Air Force Anthropometric Survey.

filename	XGLAB	YGLAB	ZGLAB	XNUC	YNUCH	ZNUCH
aout001.3	-90.33	231.32	-45.47	95.61	178.18	27.73
aout002.3	-98.59	253.21	-38.04	86.03	203.19	35.64
aout003.3	-86.96	221.95	-49.27	87.49	178.18	38.78
aout006.3	-105.94	246.95	-36.45	71.19	181.31	31.56
aout028.3	-97.37	239.14	-38.94	80.73	189.12	36.98
aout032.3	-101.56	242.26	-46.52	78.86	193.81	42.15
aout033.3	-93.43	242.26	-34.73	83.03	207.88	41.8
aout034.3	-126.23	242.26	-41.71	70.55	207.88	26.22
aout039.3	-110.31	234.45	-36.45	71.13	175.06	29.46
aout043.3	-108.02	261.02	-37.17	66.13	192.25	27.39
aout045.3	-83.72	236.01	-37.11	82.85	179.74	37.95
aout046.3	-106.98	243.83	-31.03	81.59	179.74	30.33
aout048.3	-120.41	209.44	-49.88	67.38	186	28.88
aout049.3	-120.87	240.7	-50.07	65.75	204.75	31.1
aout050.3	-102.24	251.64	-32.4	85.63	187.56	29.46
aout051.3	-97.92	223.51	-32.35	82.87	176.62	43
aout052.3	-92.59	221.95	-42.42	79.78	187.56	41.39
aout053.3	-109.06	229.76	-28.75	65.68	173.49	30.09
aout054.3	-96.54	240.7	-28	84.01	190.69	39.74
aout055.3	-100.76	243.83	-27.89	76.03	184.43	35.96
aout056.3	-85.71	237.58	-26	95.39	176.62	32.82
aout057.3	-100.09	239.14	-33.07	78.31	171.93	32.44
aout058.3	-107.67	237.58	-44.6	77.42	193.81	35.46
aout059.3	-89.97	282.9	-38.57	83.24	206.32	38.13
aout060.3	-104.56	215.69	-37.42	68.28	176.62	34.37
aout061.3	-101.95	240.7	-48.23	71.81	175.06	38.39
aout062.3	-97.07	225.07	-38.82	79.4	181.31	37.55
aout063.3	-93.11	262.58	-23.33	93.15	198.5	34.62
aout064.3	-129.32	234.45	-46.28	63.66	189.12	20.17
aout065.3	-91.33	225.07	-36.53	96.43	173.49	39.94
aout066.3	-98.85	215.69	-40.95	74.91	154.74	20.73
aout068.3	-105.46	212.57	-32	75.91	175.06	27.16
aout069.3	-130.03	245.39	-48.34	57.38	203.19	20.53
aout070.3	-104.25	234.45	-33.03	60.98	167.24	40.75
aout071.3	-88.06	271.96	-27.9	102.26	226.63	45.33
aout072.3	-99.37	221.95	-30.15	89.35	173.49	33.21
aout073.3	-80.69	240.7	-34.59	98.96	189.12	39.57
aout074.3	-102.85	242.26	-38.23	63.18	160.99	33.77
aout075.3	-128.39	225.07	-33.85	50.96	184.43	34.05
aout079.3	-106.79	218.82	-45.78	60.72	164.11	36.39
aout080.3	-95.67	239.14	-35.57	85.48	175.06	29.41
aout082.3	-89.22	243.83	-25.88	78.37	186	41.89

aout083.3	-111.14	218.82	-44.45	74	190.69	19.51
aout084.3	-93.65	237.58	-32.22	82.69	195.37	33.07
aout085.3	-92.06	232.89	-40.81	83.86	189.12	44.83
aout086.3	-96.32	239.14	-33.14	84.32	182.87	37.38
aout095.3	-80.76	239.14	-34.62	98.39	167.24	32.5
aout097.3	-101.35	264.15	-40.54	72.33	189.12	32.06
aout098.3	-119.85	279.78	-31.6	65.8	206.32	46.35
aout099.3	-106.66	270.4	-47.29	64.45	186	32.44
aout100.3	-97.96	246.95	-43.43	79.48	182.87	30.66
aout112.3	-98.99	259.46	-36.8	88.42	195.38	17.59
aout113.3	-108.47	229.76	-40.32	81.1	176.62	20.31
aout114.3	-99.44	265.71	-36.97	81.53	209.44	32.6
aout115.3	-102.23	240.7	-40.89	75.96	168.8	24.06
aout116.3	-99.74	264.15	-35.69	78.63	207.88	31.44
aout117.3	-104.11	276.65	-34.4	67.48	211	26.03
aout118.3	-108.57	231.32	-46.55	72.54	184.43	37.64
aout119.3	-99.4	262.58	-53.14	83.43	192.25	26.43
aout120.3	-105.71	254.77	-40.78	72.43	193.81	23.93
aout121.3	-102.01	245.39	-43.73	69.48	206.32	23.9
aout122.3	-91.64	253.21	-39.29	86.78	189.12	33.48
aout124.3	-97.15	246.95	-43.07	84.17	218.82	27.81
aout125.3	-99.55	246.95	-32.89	89.66	203.19	28.4
aout126.3	-103.91	251.64	-43.05	86.27	203.19	22.74
aout127.3	-103.39	282.9	-47.36	72.01	234.45	36.25
aout128.3	-97.18	267.27	-36.13	82.86	215.69	40.44
aout129.3	-92.33	254.77	-38.25	85.29	186	31.7
aout130.3	-105.19	254.77	-43.58	83.25	214.13	30.95
aout131.3	-106.5	253.21	-42.6	73.5	186	23.29
aout132.3	-92.92	270.4	-37.16	81.82	232.89	39.94
aout133.3	-107.81	271.96	-40.08	62.2	204.75	24.87
aout134.3	-84.56	279.78	-40	77.79	192.25	27.83
aout135.3	-95.5	271.96	-40.94	97.22	228.2	34.79
aout136.3	-116.21	268.84	-48.14	65.9	218.82	22.67
aout137.3	-106.29	270.4	-39.52	70.68	206.32	34.5
aout138.3	-84.4	273.52	-41.2	90.18	204.75	22.59
aout139.3	-96.34	254.77	-49.99	93.94	203.19	6.93
aout140.3	-101.24	259.46	-39.06	75.38	206.32	19.87
aout141.3	-114.71	259.46	-42.65	72.95	200.06	27.12
aout142.3	-101.94	286.03	-35.08	82.63	231.32	23.96
aout143.3	-91.1	262.58	-44.47	80.21	215.69	34.38
aout144.3	-94.71	237.58	-36.54	79.92	196.94	30.83
aout145.3	-99.68	256.33	-39.87	75.4	187.56	20.87
aout146.3	-95.13	287.59	-42.18	78.81	226.63	23.91
aout147.3	-105.79	259.46	-36.4	76.56	200.06	33.94
aout149.3	-111.38	250.08	-39.86	79.7	206.32	13.83
aout150.3	-102.88	270.4	-42.62	85.05	203.19	26.94
aout151.3	-119.26	257.89	-46.01	82.11	217.26	23.81
aout152.3	-88.57	245.39	-36.69	82.71	193.81	18.57
aout153.3	-99.1	275.09	-45.39	81.54	193.81	22.56

aout154.3	-110.48	251.64	-44.19	66.55	179.74	25.67
aout155.3	-93.81	236.01	-44.37	72.53	179.74	22.98
aout156.3	-102.52	267.27	-36.69	87.89	207.88	27.84
aout157.3	-93.02	281.34	-33.29	76.94	193.81	37.55
aout158.3	-94.48	257.89	-40.51	96.67	212.57	29.32
aout159.3	-110.17	268.84	-39.43	66.56	206.32	23.81
aout160.3	-107.55	265.71	-46.11	81.05	211	21.36
aout162.3	-90.82	281.34	-35.04	95.56	215.69	8.23
aout163.3	-96.54	262.58	-39.99	80.14	198.5	25.39
aout164.3	-104.1	251.64	-41.64	84.11	193.81	15.66
aout165.3	-109.9	254.77	-42.4	71.17	189.12	25.47
aout166.3	-110.74	282.9	-36.59	73.14	211	17.37
aout167.3	-111.23	267.27	-42.91	65.62	206.32	15.58
aout168.3	-102.59	268.84	-36.71	77.46	195.37	26.65
aout169.3	-95.06	228.2	-43.55	88.32	178.18	17.57
aout170.3	-120.34	273.52	-44.74	77.11	220.38	15.34
aout171.3	-86.43	275.09	-39.59	81.79	187.56	39.92
aout172.3	-91.99	236.01	-31.65	85.71	162.55	23.72
aout173.3	-110.89	265.71	-47.54	76.13	203.19	21.07
aout174.3	-104.72	267.27	-43.38	65.97	187.56	14.81
aout175.3	-102.56	262.58	-41.02	66.34	181.31	26.53
aout176.3	-98.92	256.33	-42.41	91.85	203.19	24.21
aout177.3	-103.63	232.89	-32.83	67.62	167.24	35.08
aout178.3	-101.54	267.27	-49.56	73.31	217.26	30.37
aout179.3	-103.23	290.72	-41.29	78.68	215.69	30.35
aout182.3	-105.63	264.15	-45.29	79.64	225.07	18.91
aout183.3	-106.47	279.78	-41.08	90.23	232.89	17.95
aout184.3	-107.33	275.09	-39.9	89.1	226.63	18.86
aout185.3	-99.18	265.71	-43.97	80.01	207.88	20.04
aout186.3	-106.49	261.02	-42.59	71.19	204.75	16.91
aout187.3	-110.08	248.52	-45.6	69.26	181.31	34.86
aout188.3	-104.72	250.08	-43.38	73.98	176.62	32.79
aout189.3	-103.41	246.95	-45.85	73.83	178.18	28.48
aout190.3	-96.59	267.27	-34.57	75.62	189.12	18.94
aout191.3	-97.2	237.58	-45.98	86.25	182.87	17.16
aout192.3	-107.69	239.14	-38.54	61.64	167.24	30.09
aout194.3	-100.47	265.71	-30.48	85.08	195.37	31.63
aout195.3	-93.19	246.95	-34.65	85.72	192.25	43.15
aout196.3	-83.16	228.2	-45.78	83.17	162.55	25.23
aout197.3	-93.15	253.21	-38.59	87.48	192.25	25.37
aout198.3	-94.52	265.71	-40.52	91.45	212.57	18.19
aout200.3	-95.12	273.52	-42.17	84.09	226.63	25.51
aout201.3	-83.52	257.89	-40.77	89.09	195.38	34.37
aout202.3	-99.37	253.21	-41.17	83.25	178.18	23.04
aout203.3	-113.52	278.21	-47.03	61.26	204.75	20.24
aout204.3	-97.18	253.21	-34.78	85.86	203.19	26.04
aout205.3	-89.85	286.03	-39.83	85.32	207.88	20.26
aout206.3	-88.59	253.21	-37.98	80.02	182.87	22.14
aout207.3	-106.33	245.39	-38.05	69.2	173.49	23.81

aout208.3	-91.85	256.33	-39.38	77.39	182.87	23.48
aout211.3	-87.02	253.21	-36.05	81.69	187.56	13.14
aout212.3	-89.34	236.01	-42.26	81.18	181.31	22.47
aout213.3	-82.24	254.77	-32.89	89.68	200.06	38.44
aout214.3	-97.71	253.21	-47.7	80.76	198.5	23.42
aout217.3	-89.79	254.77	-41.13	87.81	187.56	17.47
aout218.3	-95.25	267.27	-50.92	84.5	190.69	23.39
aout219.3	-88.16	246.95	-34.01	83.12	184.43	35.63
aout220.3	-101.45	268.84	-36.31	81.69	209.44	23.69
aout221.3	-93.31	256.33	-40.01	77.24	198.5	32
aout222.3	-98.8	245.39	-36.73	83.58	186	33.42
aout223.3	-108.97	251.64	-43.58	75.66	193.81	34.65
aout224.3	-95.53	226.63	-38.21	81.55	165.68	25.84
aout225.3	-101.05	245.39	-40.42	71.63	178.18	31.75
aout226.3	-94.88	236.01	-46.32	67.95	154.74	25.26
aout227.3	-97.44	237.58	-25.69	88.7	182.87	23.38
aout228.3	-106.17	262.58	-43.98	73.53	189.12	25.3
aout229.3	-98.44	243.83	-39.37	69.35	189.12	29.73
aout230.3	-118.14	245.39	-42.28	80.58	168.8	15
aout232.3	-104.14	232.89	-41.65	72.33	170.37	20.97
aout233.3	-98.36	254.77	-51.04	68.18	190.69	29.23
aout234.3	-98.42	251.64	-46.55	84.83	181.31	19.05
aout235.3	-99	248.52	-39.59	75.87	187.56	33.63
aout236.3	-96.62	236.01	-45.7	92.12	195.37	21.88
aout237.3	-91.75	250.08	-46.19	74.58	196.94	34.16
aout238.3	-100.84	228.2	-41.78	62.09	160.99	34.18
aout240.3	-114.97	242.26	-33.35	65.63	145.36	8.09
aout241.3	-92.7	234.45	-46.66	83.18	170.37	24.12
aout242.3	-89.85	243.83	-37.22	90.78	178.18	19.22
aout243.3	-94.57	242.26	-53.58	75.31	200.06	21.84
aout244.3	-97.37	253.21	-33.5	79.35	182.87	28.39
aout245.3	-95.21	261.02	-35.4	82.9	189.12	28.52
aout246.3	-96.01	231.32	-41.16	74.73	189.12	35.34
aout247.3	-96.74	259.46	-40.07	80.92	195.37	31.22
aout248.3	-104.51	232.89	-43.29	77.73	190.69	27.81
aout249.3	-100.92	239.14	-36.11	83.11	159.43	33.24
aout250.3	-97.04	236.01	-37.44	86.93	176.62	29.91
aout251.3	-98.5	284.47	-40.8	90	214.13	30.96
aout252.3	-102.25	246.95	-35.18	70.07	173.49	35.27
aout253.3	-92.82	228.2	-37.13	77.9	182.87	25.73
aout254.3	-93.06	259.46	-45.43	82.27	184.43	35.27
aout255.3	-104.55	273.52	-52.63	65.97	196.94	33.21
aout257.3	-105.29	228.2	-46.68	74.4	181.31	20.59
aout258.3	-106.24	243.83	-44.01	70.12	192.25	26.06
aout259.3	-96.07	232.89	-45.44	82.6	173.49	33.03
aout260.3	-97.89	246.95	-50.8	74.19	186	39.66
aout261.3	-99.58	264.15	-21.09	79.22	204.75	19.84
aout263.3	-106.6	248.52	-36.68	74.24	173.49	34.01
aout264.3	-91.11	262.58	-41.74	71.42	187.56	45.23

aout265.3	-106.31	221.95	-48.7	66.22	179.74	25.54
aout266.3	-94.6	254.77	-35.17	83.46	181.31	18.74
aout268.3	-112.26	253.21	-44.9	59.21	167.24	19.56
aout269.3	-98.88	231.32	-38.15	83.85	171.93	30
aout270.3	-101.82	243.83	-33.64	72.18	168.8	27.84
aout271.3	-108.67	256.33	-40.4	75.07	179.74	26.86
aout272.3	-116.15	237.58	-43.18	67.97	193.81	22.45
aout273.3	-110.23	250.08	-58.93	54.68	167.24	35.57
aout275.3	-114.08	243.83	-33.09	61.42	176.62	24.56
aout276.3	-101.26	267.27	-52.54	70.14	207.88	25.1
aout277.3	-98.41	253.21	-54.17	72.17	193.81	37.45
aout278.3	-94.71	231.32	-52.14	73.47	192.25	35.86
aout279.3	-104.98	257.89	-39.03	78.61	200.06	25.97
aout280.3	-112.87	257.89	-35.76	78.94	198.5	32.7
aout281.3	-103.67	264.15	-41.46	72.87	181.31	22.11
aout282.3	-106.15	251.64	-39.46	87.24	204.75	32.43
aout283.3	-102.22	262.58	-53.04	70.66	195.37	26.26
aout284.3	-102.89	251.64	-50.22	68.62	187.56	36.68
aout285.3	-110.98	278.21	-54.17	66.76	206.32	38.91
aout286.3	-111.61	246.95	-41.49	59.56	167.24	12.61
aout287.3	-91.36	257.89	-31.43	77.26	186	20.36
aout288.3	-98.63	259.46	-52.72	76.91	203.19	31.86
aout289.3	-85.84	259.46	-41.9	95.71	189.12	34.24
aout290.3	-93.23	256.33	-48.38	68.47	178.18	37.69
aout291.3	-102.98	262.58	-44.15	67.38	196.94	25.99
aout292.3	-107.46	254.77	-52.46	65.02	189.12	28.82
aout293.3	-89.1	256.33	-31.88	88.16	201.63	35.26
aout294.3	-119	248.52	-73.34	49.99	203.19	32.52
aout295.3	-112.24	248.52	-49.76	51.63	168.8	19.19
aout296.3	-107.38	250.08	-44.48	69.49	193.81	16.5
aout297.3	-105.85	250.08	-45.38	64.77	176.62	33.6
aout298.3	-114.01	268.84	-64.59	53.32	217.26	38.54
aout299.3	-108.85	275.09	-48.26	65.02	206.32	33.74
aout300.3	-119.07	239.14	-49.33	61.22	193.81	27.14
aout301.3	-106.51	271.96	-47.22	47.12	176.62	38.67
aout302.3	-98.91	261.02	-51.32	79.66	215.69	25.24
aout305.3	-113.19	254.77	-56.98	50.93	178.18	31.38
aout307.3	-108.37	273.52	-41.81	58.44	211	35.03
aout308.3	-99.31	254.77	-35.54	75.7	181.31	44.12
aout309.3	-98.53	262.58	-35.26	69.56	206.32	29.82
aout310.3	-109.67	275.09	-42.31	69.41	207.88	24.84
aout311.3	-106.37	253.21	-71.08	49.85	207.88	36.03
aout312.3	-112.41	265.71	-51.5	55.18	198.5	35.9
aout314.3	-123.28	253.21	-56.47	50.36	178.18	31.03
aout315.3	-109.68	264.15	-43.87	65.24	195.37	17.19
aout316.3	-124.67	245.39	-51.65	46.65	186	28.74
aout317.3	-114.45	245.39	-49.06	47.89	181.31	32
aout318.3	-111.81	262.58	-44.72	65.8	212.57	29.17
aout319.3	-117.48	271.96	-37.22	61.9	190.69	29.28

aout320.3	-102.71	259.46	-47.05	65.38	176.62	29.95
aout321.3	-107.52	257.89	-50.86	50.76	189.12	39.62
aout322.3	-106.52	248.52	-41.09	64.72	187.56	17.06
aout323.3	-103.37	243.83	-36.99	77.88	175.06	46.68
aout324.3	-100.64	254.77	-38.83	73.44	193.81	41.6
aout325.3	-95.15	256.33	-35.37	70.36	184.43	45.77
aout326.3	-111.01	264.15	-55.89	56.5	209.44	47.54
aout327.3	-105.61	270.4	-48.38	54.7	186	40.57
aout328.3	-123.34	259.46	-45.85	68.75	196.94	27.49
aout329.3	-99.25	242.26	-41.11	64.65	179.74	51.75
aout330.3	-99.95	256.33	-44.31	75.24	184.43	29.02
aout331.3	-109.84	257.89	-36.29	64.71	195.37	47.99
aout332.3	-113.54	273.52	-42.21	55.97	192.25	20.8
aout333.3	-123.41	256.33	-60.24	50.9	209.44	38.73
aout334.3	-108.42	254.77	-43.36	49.95	195.37	51.19
aout335.3	-114.76	259.46	-44.28	59.27	196.94	33.58
aout336.3	-132.07	265.71	-45.44	50.96	203.19	36.83
aout337.3	-118.66	275.09	-45.78	49.59	189.12	29.72
aout338.3	-101.35	215.69	-26.72	62.06	137.54	27.51
aout339.3	-105.82	271.96	-39.34	65.8	198.5	34.14
aout340.3	-95.99	259.46	-51.31	54.67	206.32	49.55
aout341.3	-105.15	254.77	-33.32	74.79	203.19	37.65
aout342.3	-106.27	256.33	-32.24	61.94	182.87	36.1
aout343.3	-91.08	253.21	-41.72	85.9	201.63	39.35
aout344.3	-110.05	281.34	-50.41	67.36	232.89	46.21
aout345.3	-112.14	261.02	-56.45	56	207.88	35.46
aout346.3	-112.52	271.96	-34.14	49.91	186	23.61
aout347.3	-104.82	240.7	-46.47	51.65	171.93	44.55
aout348.3	-103.64	262.58	-39.98	69.29	186	45.08
aout349.3	-115.16	271.96	-52.75	50.66	182.87	34.76
aout350.3	-92.63	270.4	-57.08	76.17	189.12	52.26
aout351.3	-109.91	259.46	-30.42	59.34	182.87	30.79
aout352.3	-98.12	271.96	-40.65	60.68	206.32	53.66
aout353.3	-118.68	256.33	-36.01	60.4	182.87	27.66
aout354.3	-115.18	237.58	-42.82	53.46	182.87	34.78
aout355.3	-113.76	262.58	-37.59	67.18	223.51	43.71
aout356.3	-134.2	271.96	-49.89	41.9	212.57	33.53
aout357.3	-114.9	275.09	-42.72	49.22	203.19	45.72
aout358.3	-125.01	259.46	-39.61	50.92	215.69	29.68
aout359.3	-136.25	265.71	-37.71	31.77	201.63	27.41
aout360.3	-137.33	256.33	-38.01	50.75	201.63	32.14
aout361.3	-124.07	287.59	-44.4	46.94	220.38	39.5
aout362.3	-123.19	267.27	-60.13	48.68	184.43	35.19
aout363.3	-144.51	268.84	-45.79	28.65	206.32	42.88
aout365.3	-129.83	262.58	-41.14	54.46	203.19	36.39

Output file Ultglab: This output file provides the distance in millimeters and direction angles in degrees between the Midpoint (Method 2) to the glabella.

filename	distance	phi	theta	filename	distanc e	phi	theta
a001.xyz	105.67	90	201.25	a207.xyz	103.02	90	200.48
a002.xyz	107.37	90	201.89	a208.xyz	102.09	90	198.87
a003.xyz	104.69	90	208.74	a211.xyz	97.16	90	198.6
a006.xyz	102.9	90	200.44	a212.xyz	97.2	90	200.55
a028.xyz	100.05	90	199.47	a213.xyz	97.63	90	203.45
a032.xyz	108.19	90	207.57	a214.xyz	103.74	90	201
a033.xyz	100.94	90	204.75	a217.xyz	103.64	90	200.41
a034.xyz	107.14	90	199.91	a218.xyz	104.23	90	204.2
a039.xyz	105.76	90	202.53	a219.xyz	99.98	90	199.52
a043.xyz	101.76	90	200.81	a220.xyz	105.31	90	197.3
a045.xyz	99.46	90	205.42	a221.xyz	99.58	90	202.58
a046.xyz	106.67	90	197.6	a222.xyz	107.8	90	201.02
a048.xyz	104.84	90	201.9	a223.xyz	107.67	90	202.69
a049.xyz	106.68	90	204.08	a224.xyz	101.2	90	201.32
a050.xyz	103.65	90	198.61	a225.xyz	101.92	90	202.8
a051.xyz	104.81	90	203.62	a226.xyz	101.5	90	201.86
a052.xyz	100.32	90	206.87	a227.xyz	104.7	90	195.8
a053.xyz	103.22	90	199.09	a228.xyz	105.1	90	200.43
a054.xyz	104.37	90	200.62	a229.xyz	100.46	90	204.34
a055.xyz	101.79	90	200.38	a230.xyz	112	90	197.82
a056.xyz	102.81	90	200.03	a232.xyz	103.47	90	199.16
a057.xyz	104.24	90	199.92	a233.xyz	100.54	90	204.35
a058.xyz	108.64	90	203.55	a234.xyz	104.02	90	202
a059.xyz	103.96	90	205.67	a235.xyz	100.48	90	201.84
a060.xyz	100.77	90	202.36	a236.xyz	105.44	90	200.4
a061.xyz	106.54	90	206.15	a237.xyz	99.54	90	204.16
a062.xyz	103.69	90	203.6	a238.xyz	96.8	90	205.15
a063.xyz	104.93	90	196.34	a240.xyz	107.15	90	193.15
a064.xyz	106.52	90	202.36	a241.xyz	103.67	90	202.21
a065.xyz	106.9	90	202.87	a242.xyz	102.21	90	199.18
a066.xyz	103.58	90	198.5	a243.xyz	99.25	90	204.52
a068.xyz	99.7	90	199.09	a244.xyz	103.05	90	200.56
a069.xyz	106.7	90	200.06	a245.xyz	102.68	90	200.33
a070.xyz	102.22	90	203.29	a246.xyz	101.45	90	205.15
a071.xyz	104.53	90	201.91	a247.xyz	102.4	90	201.87
a072.xyz	106.05	90	200.47	a248.xyz	100.56	90	200.58
a073.xyz	101.9	90	203.31	a249.xyz	100.99	90	199.3
a074.xyz	101.64	90	203.13	a250.xyz	106.41	90	200.8
a075.xyz	103.92	90	199.45	a251.xyz	108.75	90	198.77

a079.xyz	99.76	90	206.72	a252.xyz	100.29	90	202.09
a080.xyz	106.25	90	201.23	a253.xyz	95.62	90	202.68
a082.xyz	99.83	90	200.72	a254.xyz	108.25	90	204.15
a083.xyz	101.21	90	200.25	a255.xyz	101.68	90	206.59
a084.xyz	101.77	90	200.05	a257.xyz	101.34	90	200.07
a085.xyz	103.65	90	205.91	a258.xyz	98.86	90	202.2
a086.xyz	106.59	90	200.44	a259.xyz	103.54	90	204.41
a095.xyz	103.76	90	202.67	a260.xyz	102.02	90	206.92
a097.xyz	105.73	90	204.02	a261.xyz	98.77	90	191.27
a098.xyz	111.36	90	203.21	a263.xyz	108.48	90	200.44
a099.xyz	105.12	90	205.69	a264.xyz	101.75	90	208.09
a100.xyz	106.44	90	205.38	a265.xyz	101.8	90	202.36
a112.xyz	106.62	90	197.25	a266.xyz	102.06	90	196.41
a113.xyz	104.6	90	196.52	a268.xyz	102.12	90	199.19
a114.xyz	103.26	90	199.27	a269.xyz	103.12	90	200.19
a115.xyz	104.77	90	200.19	a270.xyz	100.58	90	199.49
a116.xyz	102.34	90	201.05	a271.xyz	104.87	90	197.85
a117.xyz	100.52	90	199.88	a272.xyz	101.99	90	200.87
a118.xyz	105.75	90	204.81	a273.xyz	105.98	90	209.44
a119.xyz	106.72	90	201.5	a275.xyz	97.88	90	198.65
a120.xyz	101.19	90	199.28	a276.xyz	100.56	90	201.84
a121.xyz	99.29	90	200.72	a277.xyz	104.76	90	204.91
a122.xyz	105.99	90	203.14	a278.xyz	101.05	90	207.31
a124.xyz	102.82	90	201.19	a279.xyz	103.36	90	200.96
a125.xyz	103.7	90	197.1	a280.xyz	108.06	90	199.87
a126.xyz	108.85	90	196.1	a281.xyz	106.11	90	200.37
a127.xyz	102.87	90	205.53	a282.xyz	110.14	90	198.97
a128.xyz	102.9	90	201.09	a283.xyz	101.88	90	203.71
a129.xyz	104.24	90	202.48	a284.xyz	104.07	90	208.59
a130.xyz	105.5	90	200.23	a285.xyz	104.16	90	206.47
a131.xyz	106.32	90	199	a286.xyz	99.41	90	197.13
a132.xyz	100.86	90	201.01	a287.xyz	98.46	90	195.74
a133.xyz	102.11	90	200.38	a288.xyz	103.59	90	204.81
a134.xyz	100.04	90	204.73	a289.xyz	101.96	90	203
a135.xyz	106.53	90	201.41	a290.xyz	100.76	90	209.43
a136.xyz	105.29	90	200.61	a291.xyz	99.57	90	200.32
a137.xyz	104.95	90	201.26	a292.xyz	104.2	90	206.08
a138.xyz	99.92	90	200.64	a293.xyz	101.33	90	201.03
a139.xyz	108.46	90	198.3	a294.xyz	104.23	90	212.8
a140.xyz	101.53	90	200.62	a295.xyz	97.45	90	202.57
a141.xyz	105.35	90	202.31	a296.xyz	102.32	90	199.44
a142.xyz	106.41	90	197.18	a297.xyz	101.49	90	202.69
a143.xyz	100.58	90	204.31	a298.xyz	101.52	90	211.39
a144.xyz	100.05	90	199.54	a299.xyz	102.25	90	205.49
a145.xyz	100.83	90	199.63	a300.xyz	103.1	90	203.62
a146.xyz	100.3	90	201.53	a301.xyz	98.87	90	209.01
a147.xyz	103.49	90	201.14	a302.xyz	102.74	90	203.24
a149.xyz	104.53	90	192.16	a305.xyz	101.79	90	207.2
a150.xyz	109.15	90	200.64	a307.xyz	99.83	90	203.88

a151.xyz	112.96	90	198.19	a308.xyz	104.36	90	202.12
a152.xyz	97.07	90	197.32	a309.xyz	98	90	200.23
a153.xyz	103.98	90	201.78	a310.xyz	103.75	90	200.24
a154.xyz	103.59	90	199.81	a311.xyz	101.75	90	215.78
a155.xyz	96.22	90	200.47	a312.xyz	99.56	90	208.61
a156.xyz	107.18	90	198.57	a314.xyz	107.63	90	206.46
a157.xyz	104.75	90	200.53	a315.xyz	98.83	90	200.33
a158.xyz	104.61	90	199.78	a316.xyz	102.71	90	205.51
a159.xyz	101.72	90	198.12	a317.xyz	97.52	90	206.24
a160.xyz	105.91	90	199.58	a318.xyz	101.67	90	202.04
a162.xyz	103.62	90	195.75	a319.xyz	104.93	90	198.52
a163.xyz	102.92	90	200.49	a320.xyz	105.56	90	202.81
a164.xyz	107.16	90	195.42	a321.xyz	103.75	90	209.12
a165.xyz	105.33	90	199.03	a322.xyz	95.62	90	198.79
a166.xyz	104.3	90	194.79	a323.xyz	107.14	90	206.48
a167.xyz	100.51	90	196.39	a324.xyz	104.61	90	204.91
a168.xyz	102.93	90	200	a325.xyz	100.59	90	203.64
a169.xyz	103.1	90	199.01	a326.xyz	102.8	90	210.53
a170.xyz	108.8	90	199.21	a327.xyz	102.43	90	208.26
a171.xyz	99.93	90	204.6	a328.xyz	109.62	90	203.22
a172.xyz	102.87	90	197.78	a329.xyz	101.66	90	210.3
a173.xyz	108.24	90	198.86	a330.xyz	101.84	90	202.39
a174.xyz	103.01	90	199.73	a331.xyz	103.92	90	206.28
a175.xyz	104.65	90	201.65	a332.xyz	102.29	90	201.14
a176.xyz	105.03	90	200.3	a333.xyz	107.66	90	210.45
a177.xyz	101.51	90	198.06	a334.xyz	99.69	90	209.06
a178.xyz	104.4	90	204.41	a335.xyz	102.35	90	204.27
a179.xyz	104.24	90	200.83	a336.xyz	106.9	90	204.49
a182.xyz	103.99	90	199.69	a337.xyz	103.5	90	203.05
a183.xyz	107.01	90	195.47	a338.xyz	97.29	90	198.12
a184.xyz	106.94	90	199.93	a339.xyz	100.84	90	201.75
a185.xyz	103.65	90	198.48	a340.xyz	97.48	90	215.91
a186.xyz	103.32	90	197.8	a341.xyz	105.11	90	200.98
a187.xyz	104.84	90	203.39	a342.xyz	97.1	90	201.14
a188.xyz	104.58	90	200.49	a343.xyz	102.95	90	205.46
a189.xyz	104.8	90	201.16	a344.xyz	106.11	90	206.26
a190.xyz	100.09	90	200.34	a345.xyz	103.67	90	210.56
a191.xyz	102.92	90	200.66	a346.xyz	99.13	90	198.8
a192.xyz	102.71	90	202	a347.xyz	99.35	90	209.96
a194.xyz	106.78	90	198.44	a348.xyz	105.04	90	205.38
a195.xyz	103.84	90	204.28	a349.xyz	104.11	90	208.5
a196.xyz	99.87	90	205.05	a350.xyz	106.78	90	213.17
a197.xyz	104.49	90	200.5	a351.xyz	99.72	90	199.88
a198.xyz	104.15	90	199.27	a352.xyz	100.57	90	209.22
a200.xyz	103.43	90	198.24	a353.xyz	103.19	90	199.45
a201.xyz	101.38	90	202.24	a354.xyz	99.14	90	204.27
a202.xyz	104.7	90	198.58	a355.xyz	102.93	90	205.07
a203.xyz	104.08	90	200.47	a356.xyz	103.52	90	205.6
a204.xyz	100.65	90	197.17	a357.xyz	102.05	90	211.63

a205.xyz	102.66	90	198.77	a358.xyz	96.81	90	200.64
a206.xyz	97.07	90	198.83	a359.xyz	102.57	90	201.7
a361.xyz	101.84	90	203.69	a363.xyz	106.57	90	206.26
a362.xyz	105.69	90	209.27	a365.xyz	106.27	90	203.47
a360.xyz	107.63	90	199.77	a361.xyz	101.84	90	203.69

APPENDIX C

Two-Sample T-tests of the Differences between Corresponding Vectors for Midpoint Method 1 and Midpoint Method 2

```
Two sample T for diff045
Midpoint        N       Mean      StDev    SE Mean
1             281      -0.95       3.65       0.22
2             281      -0.96       2.72       0.16

95% CI for mu (1) - mu (2): ( -0.52,  0.54)
T-Test mu (1) = mu (2) (vs not =): T= 0.04  P=0.97  DF=  517


Two sample T for diff067
Midpoint        N       Mean      StDev    SE Mean
1             281      -1.46       4.42       0.26
2             281      -1.44       3.23       0.19

95% CI for mu (1) - mu (2): ( -0.66,  0.62)
T-Test mu (1) = mu (2) (vs not =): T= -0.06  P=0.95  DF=  512


Two sample T for diff090
Midpoint        N       Mean      StDev    SE Mean
1             281      -1.09       4.98       0.30
2             281      -1.16       3.72       0.22

95% CI for mu (1) - mu (2): ( -0.66,  0.80)
T-Test mu (1) = mu (2) (vs not =): T= 0.19  P=0.85  DF=  518


Two sample T for diff0112
Midpoint        N       Mean      StDev    SE Mean
1             281      -0.51       5.43       0.32
2             281      -0.59       4.41       0.26

95% CI for mu (1) - mu (2): ( -0.74,  0.90)
T-Test mu (1) = mu (2) (vs not =): T= 0.19  P=0.85  DF=  537


Two sample T for diff0135
Midpoint        N       Mean      StDev    SE Mean
1             281       0.38       5.09       0.30
2             281       0.24       4.43       0.26
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```
95% CI for mu (1) - mu (2): ( -0.66,  0.92)
T-Test mu (1) = mu (2) (vs not =): T= 0.33  P=0.74  DF=  549

Two sample T for diff0157
Midpoint       N      Mean     StDev   SE Mean
1            281     -0.08      3.37      0.20
2            281      0.29      3.10      0.18

95% CI for mu (1) - mu (2): ( -0.90,  0.17)
T-Test mu (1) = mu (2) (vs not =): T= -1.34  P=0.18  DF=  556


Two sample T for diff3022
Midpoint       N      Mean     StDev   SE Mean
1            281     -1.35      2.34      0.14
2            281     -1.33      2.07      0.12

95% CI for mu (1) - mu (2): ( -0.38,  0.35)
T-Test mu (1) = mu (2) (vs not =): T= -0.09  P=0.93  DF=  551

Two sample T for diff3045
Midpoint       N      Mean     StDev   SE Mean
1            281     -2.01      3.74      0.22
2            281     -2.02      3.15      0.19

95% CI for mu (1) - mu (2): ( -0.57,  0.58)
T-Test mu (1) = mu (2) (vs not =): T= 0.03  P=0.98  DF=  544


Two sample T for diff3067
Midpoint       N      Mean     StDev   SE Mean
1            281     -1.56      4.59      0.27
2            281     -1.58      3.58      0.21

95% CI for mu (1) - mu (2): ( -0.66,  0.70)
T-Test mu (1) = mu (2) (vs not =): T= 0.05  P=0.96  DF=  528


Two sample T for diff3090
Midpoint       N      Mean     StDev   SE Mean
1            281     -1.41      5.20      0.31
2            281     -1.35      4.12      0.25

95% CI for mu (1) - mu (2): ( -0.84,  0.71)
T-Test mu (1) = mu (2) (vs not =): T= -0.16  P=0.87  DF=  532


Two sample T for diff30112
Midpoint       N      Mean     StDev   SE Mean
1            281     -1.28      5.41      0.32
2            281     -1.26      4.51      0.27

95% CI for mu (1) - mu (2): ( -0.84,  0.81)
T-Test mu (1) = mu (2) (vs not =): T= -0.03  P=0.98  DF=  542


Two sample T for diff30135
Midpoint       N      Mean     StDev   SE Mean
1            281     -0.54      4.48      0.27
2            281     -0.70      3.85      0.23

95% CI for mu (1) - mu (2): ( -0.54,  0.85)
T-Test mu (1) = mu (2) (vs not =): T= 0.44  P=0.66  DF=  547


Two sample T for diff30157
Midpoint       N      Mean     StDev   SE Mean
```

```
1           281     -0.76      2.41       0.14
2           281     -0.53      1.97       0.12

95% CI for mu (1) - mu (2): ( -0.59,  0.14)
T-Test mu (1) = mu (2) (vs not =): T= -1.21  P=0.23  DF=  538


Two sample T for diff6022
Midpoint       N       Mean      StDev   SE Mean
1           281     -1.22      2.49       0.15
2           281     -1.10      2.23       0.13

95% CI for mu (1) - mu (2): ( -0.51,  0.28)
T-Test mu (1) = mu (2) (vs not =): T= -0.58  P=0.56  DF=  553

Two sample T for diff6045
Midpoint       N       Mean      StDev   SE Mean
1           281     -1.67      3.11       0.19
2           281     -1.75      2.75       0.16

95% CI for mu (1) - mu (2): ( -0.40,  0.57)
T-Test mu (1) = mu (2) (vs not =): T= 0.34  P=0.73  DF=  551


Two sample T for diff6067
Midpoint       N       Mean      StDev   SE Mean
1           281     -2.01      3.49       0.21
2           281     -2.13      2.97       0.18
95% CI for mu (1) - mu (2): ( -0.42,  0.66)
T-Test mu (1) = mu (2) (vs not =): T= 0.43  P=0.67  DF=  546

Two sample T for diff6090
Midpoint       N       Mean      StDev   SE Mean
1           281     -2.14      3.28       0.20
2           281     -2.17      2.91       0.17

95% CI for mu (1) - mu (2): ( -0.49,  0.54)
T-Test mu (1) = mu (2) (vs not =): T= 0.10  P=0.92  DF=  552


Two sample T for diff60112
Midpoint       N       Mean      StDev   SE Mean
1           281     -1.91      3.26       0.19
2           281     -1.80      2.82       0.17

95% CI for mu (1) - mu (2): ( -0.61,  0.40)
T-Test mu (1) = mu (2) (vs not =): T= -0.43  P=0.67  DF=  548


Two sample T for diff0135
Midpoint       N       Mean      StDev   SE Mean
1           281      0.38      5.09       0.30
2           281      0.24      4.43       0.26

95% CI for mu (1) - mu (2): ( -0.66,  0.92)
T-Test mu (1) = mu (2) (vs not =): T= 0.33  P=0.74  DF=  549


Two sample T for diff60157
Midpoint       N       Mean      StDev   SE Mean
1           281     -1.01      3.64       0.22
2           281     -0.62      2.23       0.13

95% CI for mu (1) - mu (2): ( -0.88,  0.12)
T-Test mu (1) = mu (2) (vs not =): T= -1.50  P=0.13  DF=  464
```

APPENDIX D

Principal Component Analysis of Vectors from Midpoint Method 2

Eigenanalysis of the Correlation Matrix

Eigenvalue	15.700	4.438	1.709	0.939	0.285	0.196
Proportion	0.654	0.185	0.071	0.039	0.012	0.008
Cumulative	0.654	0.839	0.910	0.949	0.961	0.969
Eigenvalue	0.159	0.112	0.089	0.077	0.051	0.046
Proportion	0.007	0.005	0.004	0.003	0.002	0.002
Cumulative	0.976	0.981	0.984	0.988	0.990	0.992
Eigenvalue	0.038	0.034	0.029	0.024	0.015	0.012
Proportion	0.002	0.001	0.001	0.001	0.001	0.001
Cumulative	0.993	0.995	0.996	0.997	0.998	0.998
Eigenvalue	0.010	0.009	0.008	0.007	0.006	0.006
Proportion	0.000	0.000	0.000	0.000	0.000	0.000
Cumulative	0.999	0.999	0.999	1.000	1.000	1.000

Variable	PC1	PC2	PC3	PC4	PC5	PC6
avg00	-0.125	0.282	0.405	-0.236	0.022	0.079
avg022	-0.140	0.253	0.167	-0.515	-0.321	0.592
avg045	-0.119	0.311	-0.246	-0.411	0.299	-0.251
avg067	-0.070	0.350	-0.410	-0.169	0.310	-0.119
avg090	-0.075	0.356	-0.396	0.179	-0.063	0.175
avg0112	-0.063	0.392	-0.134	0.422	-0.349	0.105
avg0135	-0.053	0.382	0.288	0.345	-0.156	-0.207
avg0157	-0.097	0.320	0.435	0.073	0.264	-0.272
avg300	-0.245	-0.017	0.120	-0.039	-0.094	-0.153
avg3022	-0.234	-0.048	0.008	-0.177	-0.440	-0.372
avg3045	-0.239	-0.018	-0.124	-0.133	-0.244	-0.289
avg3067	-0.238	-0.014	-0.196	0.016	-0.075	-0.047
avg3090	-0.238	-0.009	-0.165	0.132	-0.025	0.130
avg30112	-0.243	0.003	-0.048	0.180	0.021	0.161
avg30135	-0.244	0.014	0.077	0.147	0.173	0.113
avg30157	-0.240	0.020	0.152	0.100	0.298	0.051
avg600	-0.241	-0.104	0.037	0.007	-0.024	-0.050
avg6022	-0.236	-0.128	-0.006	-0.052	-0.130	-0.174
avg6045	-0.239	-0.129	-0.025	-0.015	-0.046	-0.065

avg6067	-0.241	-0.125	-0.024	0.012	0.004	0.032
avg6090	-0.241	-0.122	-0.019	0.037	0.068	0.086
avg60112	-0.243	-0.111	-0.005	0.044	0.102	0.094
avg60135	-0.243	-0.101	0.020	0.058	0.158	0.146
avg60157	-0.241	-0.087	0.046	0.074	0.210	0.165

Variable	PC7	PC8	PC9	PC10	PC11	PC12
avg00	-0.362	-0.399	0.205	0.551	0.096	0.096
avg022	0.155	0.176	-0.211	-0.226	-0.090	-0.095
avg045	0.354	0.360	0.260	0.153	0.319	0.211
avg067	-0.238	-0.085	-0.203	0.023	-0.565	-0.161
avg090	-0.389	-0.133	0.062	-0.268	0.193	0.116
avg0112	-0.054	0.195	0.186	0.046	0.201	0.029
avg0135	0.210	0.318	-0.117	0.290	-0.322	-0.224
avg0157	-0.002	-0.151	-0.340	-0.445	0.324	0.006
avg300	-0.061	-0.076	0.265	-0.157	-0.200	0.177
avg3022	-0.003	-0.131	0.286	-0.252	-0.220	0.109
avg3045	0.113	-0.187	0.030	-0.021	0.134	-0.454
avg3067	0.193	-0.301	-0.296	0.151	0.194	-0.273
avg3090	0.321	-0.312	-0.169	0.140	0.110	0.127
avg30112	0.316	-0.166	0.036	0.029	-0.140	0.384
avg30135	0.176	-0.057	0.084	-0.102	-0.227	0.174
avg30157	0.040	-0.003	0.080	-0.220	-0.135	-0.004
avg600	-0.215	0.229	0.056	-0.087	0.133	-0.121
avg6022	-0.238	0.286	-0.363	0.089	0.042	0.283
avg6045	-0.204	0.183	-0.311	0.096	0.034	0.233
avg6067	-0.122	0.092	-0.140	0.153	0.015	0.072
avg6090	-0.036	0.040	-0.037	0.073	-0.037	-0.104
avg60112	-0.064	0.093	0.081	0.079	0.018	-0.123
avg60135	-0.044	0.099	0.167	0.078	0.050	-0.196
avg60157	-0.092	0.140	0.257	-0.099	0.071	-0.333

Variable	PC13	PC14	PC15	PC16	PC17	PC18
avg00	0.053	-0.096	0.030	-0.052	0.010	-0.018
avg022	-0.004	0.008	0.004	0.022	0.000	0.014
avg045	-0.051	-0.011	-0.064	0.063	0.011	-0.037
avg067	0.162	0.174	0.204	-0.099	0.060	-0.001
avg090	-0.183	-0.194	-0.499	0.042	-0.109	0.085
avg0112	0.019	0.125	0.591	0.010	0.109	-0.072
avg0135	-0.027	-0.054	-0.394	0.061	-0.086	0.018
avg0157	-0.038	0.278	0.036	-0.142	0.038	0.024
avg300	0.041	0.145	-0.019	0.549	-0.047	0.040
avg3022	-0.033	0.222	-0.053	-0.114	-0.081	-0.059
avg3045	-0.188	-0.352	0.119	-0.340	0.046	0.078
avg3067	0.085	-0.089	-0.014	0.408	-0.127	-0.150
avg3090	0.275	0.241	-0.047	0.109	0.017	0.128
avg30112	0.144	0.019	-0.069	-0.432	-0.005	-0.032
avg30135	-0.101	-0.365	-0.007	-0.128	0.103	-0.034
avg30157	-0.098	-0.455	0.310	0.282	-0.011	-0.009
avg600	0.611	-0.145	-0.188	-0.043	0.550	-0.003
avg6022	0.170	-0.185	0.109	-0.044	-0.317	0.329
avg6045	-0.159	-0.012	0.087	0.025	-0.080	-0.250
avg6067	-0.326	0.125	-0.082	-0.134	0.100	-0.472
avg6090	-0.328	0.208	-0.029	0.104	0.386	0.046
avg60112	-0.240	0.241	-0.086	0.042	0.213	0.317
avg60135	-0.079	0.176	0.062	-0.147	-0.315	0.500
avg60157	0.256	0.151	-0.062	-0.100	-0.462	-0.432

Variable	PC19	PC20	PC21	PC22	PC23	PC24

avg00	0.060	0.028	0.001	-0.049	0.038	-0.014
avg022	-0.033	-0.015	0.006	0.012	-0.013	-0.004
avg045	0.040	0.015	-0.013	0.002	0.015	0.005
avg067	-0.044	-0.005	-0.010	0.003	-0.026	0.011
avg090	0.053	-0.029	0.049	0.027	0.029	-0.029
avg0112	-0.033	0.046	-0.062	-0.031	-0.010	0.066
avg0135	0.050	-0.060	0.050	0.021	0.013	-0.075
avg0157	-0.044	0.046	-0.031	0.014	-0.021	0.043
avg300	-0.372	0.078	0.261	0.242	-0.297	0.167
avg3022	0.299	-0.074	-0.303	-0.152	0.235	-0.124
avg3045	-0.131	-0.043	0.357	0.046	-0.219	0.023
avg3067	-0.154	0.292	-0.398	0.038	0.238	-0.003
avg3090	0.322	-0.453	0.195	-0.163	-0.256	0.011
avg30112	-0.302	0.248	0.196	0.222	0.318	-0.169
avg30135	0.093	0.211	-0.393	-0.265	-0.429	0.321
avg30157	0.138	-0.354	0.102	0.029	0.367	-0.246
avg600	-0.020	-0.050	-0.139	0.140	-0.016	-0.092
avg6022	0.028	0.038	0.118	-0.143	0.203	0.393
avg6045	0.093	0.174	0.041	-0.015	-0.381	-0.627
avg6067	-0.228	-0.498	-0.159	0.215	0.065	0.314
avg6090	0.500	0.385	0.264	0.181	0.185	0.208
avg60112	-0.411	-0.033	-0.005	-0.616	0.133	-0.171
avg60135	0.049	-0.080	-0.353	0.463	-0.143	-0.116
avg60157	0.095	0.128	0.219	-0.224	0.003	0.101

Factor Analysis

Principal Component Factor Analysis of the Correlation Matrix

Unrotated Factor Loadings and Communalities

Variable	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
avg00	-0.496	0.594	0.529	-0.229	0.012	0.035
avg022	-0.553	0.533	0.218	-0.499	-0.172	0.262
avg045	-0.471	0.655	-0.322	-0.398	0.160	-0.111
avg067	-0.276	0.738	-0.536	-0.164	0.165	-0.053
avg090	-0.297	0.750	-0.518	0.173	-0.034	0.077
avg0112	-0.250	0.826	-0.176	0.409	-0.186	0.046
avg0135	-0.210	0.805	0.377	0.334	-0.083	-0.091
avg0157	-0.384	0.674	0.568	0.070	0.141	-0.120
avg300	-0.970	-0.036	0.157	-0.038	-0.050	-0.068
avg3022	-0.925	-0.102	0.010	-0.172	-0.235	-0.164
avg3045	-0.946	-0.039	-0.162	-0.129	-0.130	-0.128
avg3067	-0.944	-0.030	-0.257	0.016	-0.040	-0.021
avg3090	-0.944	-0.020	-0.215	0.128	-0.014	0.057
avg30112	-0.962	0.007	-0.063	0.175	0.011	0.071
avg30135	-0.968	0.030	0.101	0.143	0.092	0.050
avg30157	-0.950	0.043	0.199	0.097	0.159	0.023
avg600	-0.955	-0.220	0.048	0.007	-0.013	-0.022
avg6022	-0.934	-0.270	-0.008	-0.051	-0.070	-0.077
avg6045	-0.946	-0.272	-0.033	-0.014	-0.024	-0.029
avg6067	-0.954	-0.263	-0.032	0.012	0.002	0.014
avg6090	-0.956	-0.257	-0.025	0.036	0.036	0.038
avg60112	-0.961	-0.234	-0.006	0.042	0.054	0.041
avg60135	-0.961	-0.213	0.026	0.056	0.084	0.064
avg60157	-0.956	-0.184	0.060	0.072	0.112	0.073
Variance	15.700	4.438	1.709	0.939	0.285	0.196

% Var	0.654	0.185	0.071	0.039	0.012	0.008

Variable	Factor7	Factor8	Factor9	Factor10	Factor11	Factor12
avg00	-0.144	-0.134	0.061	0.153	0.022	0.021
avg022	0.062	0.059	-0.063	-0.063	-0.020	-0.020
avg045	0.141	0.121	0.078	0.042	0.072	0.046
avg067	-0.095	-0.029	-0.061	0.006	-0.128	-0.035
avg090	-0.155	-0.044	0.018	-0.074	0.044	0.025
avg0112	-0.022	0.065	0.056	0.013	0.045	0.006
avg0135	0.084	0.106	-0.035	0.080	-0.073	-0.048
avg0157	-0.001	-0.051	-0.102	-0.123	0.073	0.001
avg300	-0.024	-0.026	0.079	-0.044	-0.045	0.038
avg3022	-0.001	-0.044	0.086	-0.070	-0.050	0.024
avg3045	0.045	-0.063	0.009	-0.006	0.030	-0.098
avg3067	0.077	-0.101	-0.089	0.042	0.044	-0.059
avg3090	0.128	-0.105	-0.050	0.039	0.025	0.027
avg30112	0.126	-0.056	0.011	0.008	-0.032	0.083
avg30135	0.070	-0.019	0.025	-0.028	-0.051	0.037
avg30157	0.016	-0.001	0.024	-0.061	-0.031	-0.001
avg600	-0.086	0.077	0.017	-0.024	0.030	-0.026
avg6022	-0.095	0.096	-0.109	0.025	0.010	0.061
avg6045	-0.082	0.061	-0.093	0.027	0.008	0.050
avg6067	-0.049	0.031	-0.042	0.042	0.003	0.015
avg6090	-0.014	0.013	-0.011	0.020	-0.008	-0.022
avg60112	-0.025	0.031	0.024	0.022	0.004	-0.026
avg60135	-0.017	0.033	0.050	0.022	0.011	-0.042
avg60157	-0.037	0.047	0.077	-0.027	0.016	-0.072
Variance	0.159	0.112	0.089	0.077	0.051	0.046
% Var	0.007	0.005	0.004	0.003	0.002	0.002

Variable	Factor13	Factor14	Factor15	Factor16	Factor17	Factor18
avg00	0.010	-0.018	0.005	-0.008	0.001	-0.002
avg022	-0.001	0.001	0.001	0.003	0.000	0.002
avg045	-0.010	-0.002	-0.011	0.010	0.001	-0.004
avg067	0.032	0.032	0.035	-0.015	0.007	-0.000
avg090	-0.036	-0.036	-0.085	0.006	-0.014	0.010
avg0112	0.004	0.023	0.101	0.002	0.013	-0.008
avg0135	-0.005	-0.010	-0.067	0.009	-0.011	0.002
avg0157	-0.007	0.051	0.006	-0.022	0.005	0.003
avg300	0.008	0.027	-0.003	0.084	-0.006	0.004
avg3022	-0.007	0.041	-0.009	-0.018	-0.010	-0.007
avg3045	-0.037	-0.065	0.020	-0.052	0.006	0.009
avg3067	0.017	-0.016	-0.002	0.063	-0.016	-0.017
avg3090	0.054	0.044	-0.008	0.017	0.002	0.014
avg30112	0.028	0.003	-0.012	-0.066	-0.001	-0.004
avg30135	-0.020	-0.067	-0.001	-0.020	0.013	-0.004
avg30157	-0.019	-0.084	0.053	0.043	-0.001	-0.001
avg600	0.119	-0.027	-0.032	-0.007	0.068	-0.000
avg6022	0.033	-0.034	0.019	-0.007	-0.039	0.037
avg6045	-0.031	-0.002	0.015	0.004	-0.010	-0.028
avg6067	-0.063	0.023	-0.014	-0.021	0.012	-0.053
avg6090	-0.064	0.038	-0.005	0.016	0.048	0.005
avg60112	-0.047	0.044	-0.015	0.006	0.026	0.035
avg60135	-0.015	0.032	0.011	-0.023	-0.039	0.056
avg60157	0.050	0.028	-0.011	-0.015	-0.057	-0.048
Variance	0.038	0.034	0.029	0.024	0.015	0.012
% Var	0.002	0.001	0.001	0.001	0.001	0.001

Variable	Factor19	Factor20	Factor21	Factor22	Factor23	Factor24
avg00	0.006	0.003	0.000	-0.004	0.003	-0.001
avg022	-0.003	-0.001	0.001	0.001	-0.001	-0.000
avg045	0.004	0.001	-0.001	0.000	0.001	0.000
avg067	-0.004	-0.000	-0.001	0.000	-0.002	0.001
avg090	0.005	-0.003	0.005	0.002	0.002	-0.002
avg0112	-0.003	0.004	-0.006	-0.003	-0.001	0.005
avg0135	0.005	-0.006	0.005	0.002	0.001	-0.006
avg0157	-0.004	0.004	-0.003	0.001	-0.002	0.003
avg300	-0.038	0.007	0.024	0.020	-0.023	0.013
avg3022	0.030	-0.007	-0.028	-0.012	0.018	-0.009
avg3045	-0.013	-0.004	0.033	0.004	-0.017	0.002
avg3067	-0.016	0.028	-0.037	0.003	0.018	-0.000
avg3090	0.032	-0.043	0.018	-0.013	-0.020	0.001
avg30112	-0.030	0.024	0.018	0.018	0.024	-0.013
avg30135	0.009	0.020	-0.036	-0.022	-0.033	0.024
avg30157	0.014	-0.034	0.009	0.002	0.028	-0.019
avg600	-0.002	-0.005	-0.013	0.012	-0.001	-0.007
avg6022	0.003	0.004	0.011	-0.012	0.016	0.030
avg6045	0.009	0.017	0.004	-0.001	-0.029	-0.048
avg6067	-0.023	-0.047	-0.015	0.018	0.005	0.024
avg6090	0.050	0.037	0.024	0.015	0.014	0.016
avg60112	-0.041	-0.003	-0.000	-0.051	0.010	-0.013
avg60135	0.005	-0.008	-0.033	0.038	-0.011	-0.009
avg60157	0.010	0.012	0.020	-0.018	0.000	0.008
Variance	0.010	0.009	0.008	0.007	0.006	0.006
% Var	0.000	0.000	0.000	0.000	0.000	0.000

Variable	Communality
avg00	1.000
avg022	1.000
avg045	1.000
avg067	1.000
avg090	1.000
avg0112	1.000
avg0135	1.000
avg0157	1.000
avg300	1.000
avg3022	1.000
avg3045	1.000
avg3067	1.000
avg3090	1.000
avg30112	1.000
avg30135	1.000
avg30157	1.000
avg600	1.000
avg6022	1.000
avg6045	1.000
avg6067	1.000
avg6090	1.000
avg60112	1.000
avg60135	1.000
avg60157	1.000
Variance	24.000
% Var	1.000

Rotated Factor Loadings and Communalities
Varimax Rotation

Variable	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
avg00	0.257	-0.148	-0.780	0.426	0.087	-0.038
avg022	0.313	-0.287	-0.429	0.796	0.000	-0.042
avg045	0.234	-0.732	-0.204	0.371	0.087	-0.469
avg067	0.055	-0.947	-0.121	0.154	0.164	-0.098
avg090	0.096	-0.952	-0.188	0.003	-0.094	0.125
avg0112	0.034	-0.738	-0.508	-0.073	-0.416	0.126
avg0135	-0.022	-0.334	-0.893	-0.012	-0.200	0.023
avg0157	0.154	-0.162	-0.943	0.141	0.104	-0.050
avg300	0.923	-0.046	-0.270	0.170	0.015	-0.040
avg3022	0.889	-0.075	-0.091	0.240	-0.007	-0.058
avg3045	0.903	-0.237	-0.032	0.186	0.007	-0.092
avg3067	0.915	-0.313	-0.001	0.083	-0.005	-0.033
avg3090	0.919	-0.292	-0.051	0.032	-0.068	-0.003
avg30112	0.927	-0.217	-0.180	0.030	-0.087	0.000
avg30135	0.920	-0.134	-0.307	0.057	-0.014	-0.017
avg30157	0.893	-0.081	-0.375	0.078	0.044	-0.040
avg600	0.973	0.002	-0.085	0.092	0.012	-0.011
avg6022	0.965	0.009	-0.007	0.109	0.031	-0.021
avg6045	0.981	-0.012	0.004	0.084	0.032	-0.006
avg6067	0.988	-0.018	-0.005	0.075	0.023	-0.005
avg6090	0.990	-0.018	-0.018	0.064	0.020	-0.007
avg60112	0.989	-0.024	-0.046	0.061	0.014	-0.020
avg60135	0.982	-0.019	-0.081	0.061	0.005	-0.025
avg60157	0.969	-0.020	-0.124	0.057	0.016	-0.015
Variance	14.578	3.447	3.211	1.190	0.293	0.286
% Var	0.607	0.144	0.134	0.050	0.012	0.012

Variable	Factor7	Factor8	Factor9	Factor10	Factor11	Factor12
avg00	0.031	0.334	-0.013	0.001	0.013	0.002
avg022	0.019	-0.008	0.003	0.003	0.001	0.004
avg045	0.034	0.015	0.002	0.002	0.005	0.009
avg067	0.004	0.022	0.004	0.011	-0.008	0.008
avg090	-0.004	0.001	0.007	-0.014	0.010	-0.001
avg0112	0.004	-0.040	0.017	-0.006	-0.012	-0.001
avg0135	0.001	-0.095	0.007	0.001	-0.202	0.003
avg0157	-0.004	-0.049	0.003	-0.005	0.158	-0.004
avg300	0.141	0.051	-0.017	-0.016	0.010	-0.038
avg3022	0.364	0.022	-0.007	0.018	-0.001	0.014
avg3045	0.197	0.009	0.026	0.006	-0.003	0.210
avg3067	0.039	-0.017	0.139	0.000	-0.006	0.085
avg3090	-0.015	-0.027	0.239	-0.032	-0.002	0.011
avg30112	-0.029	-0.029	0.113	-0.049	-0.012	-0.016
avg30135	-0.058	-0.014	0.024	-0.064	-0.009	-0.017
avg30157	-0.079	0.005	-0.021	-0.047	0.034	-0.013
avg600	0.027	0.022	-0.054	0.034	0.008	-0.008
avg6022	0.064	0.004	-0.044	0.217	-0.004	0.005
avg6045	0.020	0.010	-0.040	0.110	0.011	-0.009
avg6067	-0.006	0.024	-0.026	0.019	-0.002	-0.005
avg6090	-0.029	0.011	-0.010	-0.042	0.004	-0.010
avg60112	-0.041	0.024	-0.030	-0.048	0.005	-0.017
avg60135	-0.068	0.036	-0.027	-0.052	0.012	-0.018
avg60157	-0.078	0.025	-0.045	-0.071	0.022	-0.033
Variance	0.224	0.135	0.103	0.083	0.069	0.056

% Var	0.009	0.006	0.004	0.003	0.003	0.002

Variable	Factor13	Factor14	Factor15	Factor16	Factor17	Factor18
avg00	-0.006	-0.003	0.000	-0.003	-0.003	0.003
avg022	0.001	-0.001	0.002	-0.001	0.002	-0.000
avg045	0.000	-0.007	0.003	-0.001	0.002	0.001
avg067	-0.005	-0.158	0.005	0.006	0.005	-0.002
avg090	0.010	0.154	0.001	-0.004	-0.000	0.001
avg0112	0.009	0.009	-0.003	0.001	0.001	-0.000
avg0135	0.011	-0.005	-0.016	0.004	0.003	-0.007
avg0157	0.000	0.007	0.019	-0.004	-0.002	0.008
avg300	-0.006	0.004	0.035	-0.004	-0.002	0.000
avg3022	-0.005	-0.001	-0.013	-0.005	0.001	-0.006
avg3045	-0.012	-0.003	-0.009	0.003	0.034	-0.012
avg3067	-0.007	-0.004	-0.017	0.017	0.165	-0.019
avg3090	0.044	0.001	-0.007	0.017	0.021	-0.009
avg30112	0.186	0.008	0.018	0.023	-0.004	-0.004
avg30135	0.086	0.012	0.091	0.020	-0.012	-0.006
avg30157	0.018	-0.004	0.183	0.007	-0.012	0.013
avg600	-0.030	0.007	-0.011	-0.175	-0.015	0.018
avg6022	-0.032	-0.009	-0.029	-0.022	-0.001	-0.020
avg6045	-0.033	-0.004	-0.034	0.019	0.004	-0.042
avg6067	-0.018	0.005	-0.040	0.036	-0.008	-0.036
avg6090	-0.028	-0.003	-0.017	0.042	-0.011	-0.037
avg60112	-0.022	0.004	-0.021	0.024	-0.021	-0.014
avg60135	-0.009	-0.003	0.003	0.019	-0.026	0.030
avg60157	-0.009	0.003	0.018	-0.026	-0.024	0.155
Variance	0.050	0.050	0.049	0.038	0.031	0.031
% Var	0.002	0.002	0.002	0.002	0.001	0.001

Variable	Factor19	Factor20	Factor21	Factor22	Factor23	Factor24
avg00	0.003	-0.001	-0.001	0.001	0.000	0.000
avg022	0.001	0.001	0.000	0.000	0.000	-0.000
avg045	0.001	0.000	-0.000	-0.000	-0.000	0.000
avg067	-0.001	-0.001	-0.000	-0.001	0.000	-0.001
avg090	0.002	0.005	0.001	0.000	-0.001	0.000
avg0112	-0.001	0.001	-0.000	-0.001	-0.001	0.000
avg0135	-0.002	0.002	0.001	-0.000	-0.001	-0.000
avg0157	0.004	0.001	-0.002	-0.001	0.000	0.000
avg300	0.131	-0.002	0.006	-0.008	-0.004	0.001
avg3022	0.004	-0.002	0.002	0.001	0.002	-0.001
avg3045	-0.009	-0.003	0.002	-0.000	-0.001	-0.001
avg3067	-0.001	-0.004	0.007	-0.001	0.001	-0.002
avg3090	-0.003	0.004	0.001	-0.001	-0.003	-0.001
avg30112	-0.002	0.011	0.001	-0.002	-0.003	-0.001
avg30135	-0.003	0.113	0.003	-0.007	-0.006	-0.003
avg30157	0.014	0.011	-0.002	-0.004	-0.004	-0.001
avg600	0.002	-0.006	0.004	-0.005	-0.003	-0.002
avg6022	-0.003	-0.008	0.003	0.002	-0.000	-0.001
avg6045	-0.013	-0.011	0.027	0.007	0.091	-0.011
avg6067	-0.023	-0.014	0.023	0.094	0.005	-0.008
avg6090	-0.019	-0.016	0.034	-0.042	-0.026	-0.050
avg60112	-0.002	-0.014	0.004	-0.019	-0.025	0.077
avg60135	-0.012	-0.005	-0.104	-0.016	-0.018	0.001
avg60157	-0.000	-0.003	-0.010	-0.007	-0.008	-0.001
Variance	0.019	0.014	0.013	0.012	0.010	0.009
% Var	0.001	0.001	0.001	0.000	0.000	0.000

Variable	Communality
avg00	1.000
avg022	1.000
avg045	1.000
avg067	1.000
avg090	1.000
avg0112	1.000
avg0135	1.000
avg0157	1.000
avg300	1.000
avg3022	1.000
avg3045	1.000
avg3067	1.000
avg3090	1.000
avg30112	1.000
avg30135	1.000
avg30157	1.000
avg600	1.000
avg6022	1.000
avg6045	1.000
avg6067	1.000
avg6090	1.000
avg60112	1.000
avg60135	1.000
avg60157	1.000
Variance	24.000
% Var	1.000

Factor Score Coefficients

Variable	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
avg00	-0.045	0.062	-0.221	-0.053	-0.060	0.002
avg022	-0.067	0.074	0.172	1.445	-0.253	0.799
avg045	-0.037	-0.079	0.061	-0.090	-0.619	-2.279
avg067	-0.004	-0.494	0.074	-0.096	1.027	0.898
avg090	-0.013	-0.505	0.081	-0.084	1.167	0.840
avg0112	-0.005	-0.099	0.056	-0.015	-2.599	-0.302
avg0135	0.004	0.064	-0.474	-0.237	0.693	-0.016
avg0157	-0.022	0.080	-0.590	-0.298	0.691	0.006
avg300	0.058	0.001	0.013	-0.015	-0.023	0.007
avg3022	0.016	-0.005	0.000	-0.056	0.131	0.147
avg3045	0.042	0.014	-0.002	-0.022	-0.017	0.052
avg3067	0.052	0.012	-0.003	-0.021	-0.027	-0.016
avg3090	0.042	0.014	0.001	-0.020	0.103	0.000
avg30112	0.054	0.008	0.012	-0.010	0.042	0.001
avg30135	0.059	0.012	0.010	-0.014	-0.047	-0.018
avg30157	0.035	0.008	0.005	-0.020	-0.043	0.010
avg600	0.085	-0.004	0.007	-0.024	0.004	0.005
avg6022	0.068	0.001	0.002	-0.034	-0.035	-0.002
avg6045	0.092	-0.002	-0.001	-0.030	-0.005	-0.008
avg6067	0.101	-0.001	-0.002	-0.028	-0.005	-0.005
avg6090	0.108	-0.000	-0.002	-0.025	-0.001	-0.012
avg60112	0.102	-0.001	0.001	-0.024	0.001	0.002
avg60135	0.094	-0.003	0.006	-0.020	0.009	0.010
avg60157	0.080	-0.001	0.012	-0.020	-0.008	0.009

Variable	Factor7	Factor8	Factor9	Factor10	Factor11	Factor12

avg00	-0.064	2.671	0.444	0.305	-0.273	0.091
avg022	-0.536	-1.153	-0.041	-0.147	-0.092	0.054
avg045	-0.227	0.180	-0.083	0.137	0.112	-0.639
avg067	0.161	-0.127	-0.258	-0.196	-0.277	-0.143
avg090	0.176	-0.099	-0.156	0.143	-0.659	-0.115
avg0112	-0.141	0.640	-0.681	0.160	2.163	-0.033
avg0135	0.092	-0.846	0.169	-0.131	-3.271	0.046
avg0157	0.163	-1.227	0.103	-0.019	2.527	0.189
avg300	-0.248	-0.053	-0.206	-0.267	0.010	0.385
avg3022	3.034	-0.169	0.170	-0.670	-0.138	-3.069
avg3045	-0.297	-0.038	-0.265	-0.210	0.067	5.502
avg3067	-0.080	0.034	-0.850	-0.295	0.078	-1.193
avg3090	-0.021	0.081	5.219	0.479	-0.291	-0.176
avg30112	-0.145	0.060	-1.376	0.162	0.186	0.199
avg30135	-0.113	0.025	-0.334	-0.189	0.127	-0.076
avg30157	0.105	0.043	0.223	0.338	-0.544	-0.241
avg600	-0.250	-0.036	-0.205	-1.181	-0.025	-0.148
avg6022	-0.291	0.030	0.228	4.912	0.019	0.026
avg6045	-0.246	-0.019	-0.315	-0.467	-0.017	-0.081
avg6067	-0.238	-0.051	-0.451	-0.753	0.080	-0.210
avg6090	-0.255	-0.013	-0.625	-0.862	0.115	-0.338
avg60112	-0.217	-0.029	-0.492	-0.686	0.061	-0.261
avg60135	-0.193	-0.055	-0.430	-0.569	-0.000	-0.201
avg60157	-0.119	-0.024	-0.141	0.220	-0.172	0.062

Variable	Factor13	Factor14	Factor15	Factor16	Factor17	Factor18
avg00	0.252	0.095	-0.042	0.020	0.225	0.075
avg022	-0.042	-0.128	0.101	-0.023	-0.042	-0.036
avg045	-0.116	1.093	-0.118	-0.047	0.094	-0.093
avg067	0.232	-3.070	-0.156	-0.182	-0.476	-0.033
avg090	0.047	3.579	0.205	0.112	-0.418	0.076
avg0112	-0.569	-1.956	0.154	0.112	0.392	-0.201
avg0135	-0.430	0.518	-0.274	0.035	-0.193	0.231
avg0157	0.063	0.067	-1.437	0.054	0.042	-0.152
avg300	-0.160	-0.118	-1.024	0.396	-0.420	-0.425
avg3022	-0.274	-0.009	1.270	0.373	0.803	0.577
avg3045	0.302	-0.020	-0.370	0.210	-2.746	0.297
avg3067	0.236	0.028	-0.069	-0.101	7.186	0.044
avg3090	-3.328	0.135	0.516	-0.359	-4.470	0.514
avg30112	6.404	-0.183	-0.737	-0.252	0.891	-0.245
avg30135	-0.662	-0.200	-0.890	0.182	0.078	-0.347
avg30157	-0.582	0.194	6.118	-0.065	0.142	-0.669
avg600	0.050	-0.270	-0.391	-5.713	0.112	-2.149
avg6022	0.425	0.296	0.432	1.406	-0.342	1.607
avg6045	-0.282	0.003	-0.501	0.426	-0.389	-0.552
avg6067	-0.457	-0.107	-0.604	0.358	-0.190	-0.868
avg6090	-0.597	0.001	-0.902	0.957	-0.454	-1.540
avg60112	-0.466	-0.060	-0.675	0.635	-0.248	-1.105
avg60135	-0.345	-0.001	-0.660	0.262	0.019	-1.513
avg60157	-0.234	-0.039	-0.783	1.298	0.105	6.532

Variable	Factor19	Factor20	Factor21	Factor22	Factor23	Factor24
avg00	-0.887	0.018	0.466	-0.371	0.026	0.036
avg022	0.253	-0.048	-0.164	0.088	0.040	0.127
avg045	-0.172	-0.019	0.300	0.042	0.058	-0.302
avg067	0.218	0.207	-0.135	0.126	-0.081	0.141
avg090	0.008	-0.505	-0.394	-0.309	-0.065	-0.134
avg0112	-0.006	0.640	0.485	0.261	-0.303	-0.052
avg0135	-0.215	-0.947	-0.284	-0.116	0.487	-0.002

avg0157	-0.154	0.111	-0.242	0.094	-0.352	-0.017
avg300	7.966	0.413	-0.254	0.588	0.094	-1.180
avg3022	-4.492	-0.277	-0.431	-0.207	-0.121	0.742
avg3045	1.699	0.127	0.244	-0.149	0.676	-0.166
avg3067	-0.625	-0.021	-0.485	-0.055	-0.775	0.255
avg3090	0.119	0.632	0.096	0.461	0.835	0.427
avg30112	0.046	-4.232	0.250	-0.669	-0.013	-0.263
avg30135	-0.050	9.468	0.417	-0.042	-0.233	0.062
avg30157	-1.974	-4.616	0.317	0.655	0.202	0.600
avg600	-0.525	-0.207	-0.131	0.313	0.410	-0.520
avg6022	0.231	0.812	0.073	-0.863	-5.479	0.252
avg6045	-0.145	-0.224	0.005	-2.233	10.023	0.421
avg6067	-0.050	-0.194	0.592	8.613	-2.204	-1.355
avg6090	-0.998	-1.224	3.033	-4.125	-3.351	-6.020
avg60112	-0.819	-0.552	2.156	-2.121	-1.108	9.024
avg60135	0.003	-0.224	-8.704	-0.117	0.732	-1.839
avg60157	-0.194	0.337	2.742	0.036	0.388	-0.342

APPENDIX E

One-Way Analysis of Variance and Two-Sample T-Tests of Deltas for Different Sizing Methods

Nomenclature:

d-br-km = difference between head breadth and design helmet breadth when sizing was completed by k-means clustering of head breadth and head length.

d-br-pca = difference head breadth and design helmet breadth when sizing was completed by k-means clustering of PCA scores.

d-br-cut = difference between head breadth and design helmet breadth when sizing was completed by traditional reference head method.

d-l-km = difference between head length and design helmet length when sizing was completed by k-means clustering of head breadth and head length.

d-l-pca = difference between head length and design helmet length when sizing was completed by k-means clustering of PCA scores.

d-l-cut = difference between head length and design helmet length when sizing was completed by traditional reference head method.

<u>One-Way Analysis of Variance</u>

Analysis of Variance

Source	DF	SS	MS	F	P
Factor	2	5097.0	2548.5	222.78	0.000
Error	834	9540.5	11.4		
Total	836	14637.5			

```
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  --+---------+---------+---------+----
d-br-km   281     4.126     2.513   (-*-)
d-br-pca  281     9.544     4.438                              (-*-)
d-br-cut  275     4.539     2.870     (-*-)
                                   --+---------+---------+---------+----
Pooled StDev =    3.382            4.0       6.0       8.0      10.0
```

<u>Two Sample T-Test and Confidence Interval</u>

Two sample T for d-br-km vs d-br-cut

	N	Mean	StDev	SE Mean
d-br-km	281	4.13	2.51	0.15

```
d-br-cut  275      4.54      2.87      0.17

90% CI for mu d-br-km - mu d-br-cut: ( -0.79,  -0.03)
T-Test mu d-br-km = mu d-br-cut (vs not =): T= -1.80  P=0.072  DF=  541
```

One-Way Analysis of Variance

```
Analysis of Variance
Source     DF        SS        MS        F        P
Factor      2    6321.4    3160.7   131.56    0.000
Error     834   20036.0      24.0
Total     836   26357.4
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  ------+---------+---------+---------+
d-l-km    281     5.389     3.756  (--*--)
d-l-pca   281    11.456     6.632                                (--*--)
d-l-cut   275     5.926     3.709     (--*--)
                                   ------+---------+---------+---------+
Pooled StDev =    4.901                 6.0       8.0      10.0      12.0
```

Two Sample T-Test and Confidence Interval

```
Two sample T for d-l-km vs d-l-cut
            N      Mean     StDev   SE Mean
d-l-km    281      5.39      3.76      0.22
d-l-cut   275      5.93      3.71      0.22

90% CI for mu d-l-km - mu d-l-cut: ( -1.06,  -0.02)
T-Test mu d-l-km = mu d-l-cut (vs not =): T= -1.70  P=0.090  DF=  553
```

APPENDIX F

Variance Comparison Test between the Differences of Corresponding Vectors for Midpoint Method 1 and Midpoint Method 2.

Nomenclature:
Midpoint Method 1 = Factor 1
Midpoint Method 2 = Factor 2
Diffxxxx = The difference between corresponding vectors at the associated direction angles.

```
Homogeneity of Variance
Response     Diff022
Factors      Midpoint
ConfLvl      95.0000
Bonferroni confidence intervals for standard deviations
  Lower      Sigma      Upper    N  Factor Levels
 2.15580   2.36051   2.60640   281   1
 1.69836   1.85963   2.05335   281   2
Bartlett's Test (normal distribution)
Test Statistic: 15.750
P-Value       :  0.000
Levene's Test (any continuous distribution)
Test Statistic: 18.548
P-Value       :  0.000

 Homogeneity of Variance
Response     diff045
Factors      Midpoint
ConfLvl      95.0000
Bonferroni confidence intervals for standard deviations
  Lower      Sigma      Upper    N  Factor Levels
3.33463   3.65128   4.03164   281   1
 2.48176   2.71742   3.00049   281   2
Bartlett's Test (normal distribution)
Test Statistic: 24.042
P-Value       :  0.000
Levene's Test (any continuous distribution)
Test Statistic: 22.429
P-Value       :  0.000
```

```
Homogeneity of Variance
Response    diff067
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower      Sigma      Upper   N  Factor Levels
 4.03497   4.41812   4.87836  281  1
 2.94668   3.22649   3.56260  281  2
Bartlett's Test (normal distribution)
Test Statistic: 27.171
P-Value        :  0.000
Levene's Test (any continuous distribution)
Test Statistic: 23.477
P-Value        :  0.000

Homogeneity of Variance
Response    diff090
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower      Sigma      Upper   N  Factor Levels
 4.54366   4.97512   5.49338  281  1
 3.39512   3.71751   4.10476  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 23.404
P-Value        :  0.000
 Levene's Test (any continuous distribution)
 Test Statistic: 22.949
P-Value        :  0.000

 Homogeneity of Variance
Response    diff0112
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
   Lower      Sigma      Upper   N  Factor Levels
 4.96065   5.43171   5.99753  281  1
 4.03204   4.41491   4.87482  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 11.922
P-Value        :  0.001
 Levene's Test (any continuous distribution)
 Test Statistic: 11.844
P-Value        :  0.001

Homogeneity of Variance
Response    diff0135
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower      Sigma      Upper   N  Factor Levels
 4.64757   5.08890   5.61901  281  1
 4.04308   4.42701   4.88817  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 5.409
P-Value        : 0.020
 Levene's Test (any continuous distribution)
 Test Statistic: 4.308
P-Value        : 0.038

Homogeneity of Variance
Response    diff0157
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower      Sigma      Upper   N  Factor Levels
 3.07895   3.37132   3.72251  281  1
```

```
 2.83128   3.10014   3.42308  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 1.963
P-Value        : 0.161
 Levene's Test (any continuous distribution)
 Test Statistic: 0.081
P-Value        : 0.776

Homogeneity of Variance
Response    diff3022
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower     Sigma     Upper   N  Factor Levels
 2.13861   2.34169   2.58562  281  1
 1.89133   2.07093   2.28665  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 4.209
P-Value        : 0.040
 Levene's Test (any continuous distribution)
 Test Statistic: 3.282
P-Value        : 0.071

Homogeneity of Variance
Response    diff3045
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower     Sigma     Upper   N  Factor Levels
 3.41980   3.74454   4.13461  281  1
 2.87936   3.15278   3.48121  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 8.229
P-Value        : 0.004
 Levene's Test (any continuous distribution)
 Test Statistic: 8.307
P-Value        : 0.004

Homogeneity of Variance
Response    diff3067
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower     Sigma     Upper   N  Factor Levels
 4.19237   4.59047   5.06866  281  1
 3.26548   3.57556   3.94803  281  2
Bartlett's Test (normal distribution)
Test Statistic: 17.271
P-Value        :  0.000
Levene's Test (any continuous distribution)
 Test Statistic: 19.150
P-Value        :  0.000

Homogeneity of Variance
Response    diff3090
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower     Sigma     Upper   N  Factor Levels
 4.75130   5.20248   5.74442  281  1
 3.76393   4.12135   4.55067  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 15.033
P-Value        :  0.000
Levene's Test (any continuous distribution)
Test Statistic: 13.853
P-Value        :  0.000
```

```
Homogeneity of Variance
Response    diff30112
Factors     Midpoint
ConfLvl     95.0000
 Bonferroni confidence intervals for standard deviations
   Lower     Sigma     Upper   N  Factor Levels
 4.93699   5.40580   5.96892  281  1
 4.12007   4.51130   4.98125  281  2
 Bartlett's Test (normal distribution)
 Test Statistic: 9.096
P-Value        : 0.003
Levene's Test (any continuous distribution)
Test Statistic: 6.291
P-Value        : 0.012

Homogeneity of Variance
Response    diff30135
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper   N  Factor Levels
 4.08763   4.47578   4.94202  281  1
 3.51531   3.84911   4.25008  281  2
Bartlett's Test (normal distribution)
Test Statistic: 6.335
P-Value        : 0.012
Levene's Test (any continuous distribution)
Test Statistic: 3.625
P-Value        : 0.057

Homogeneity of Variance
Response    diff30157
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper   N  Factor Levels
 2.19811   2.40684   2.65756  281  1
 1.79465   1.96506   2.16976  281  2
Bartlett's Test (normal distribution)
Test Statistic: 11.416
P-Value        :  0.001
 Levene's Test (any continuous distribution)
Test Statistic: 7.373
P-Value        : 0.007

Homogeneity of Variance
Response    diff6022
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper   N  Factor Levels
 2.27003   2.48559   2.74451  281  1
 2.03489   2.22812   2.46023  281  2
Bartlett's Test (normal distribution)
Test Statistic: 3.335
P-Value        : 0.068
Levene's Test (any continuous distribution)
Test Statistic: 2.597
P-Value        : 0.108

Homogeneity of Variance
Response    diff6045
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper   N  Factor Levels
```

```
 2.83992   3.10959   3.43352  281  1
 2.51075   2.74917   3.03555  281  2
Bartlett's Test (normal distribution)
Test Statistic: 4.231
P-Value       : 0.040
Levene's Test (any continuous distribution)
Test Statistic: 2.101
P-Value       : 0.148

Homogeneity of Variance
Response    diff6067
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper    N  Factor Levels
 3.18648   3.48906   3.85252  281  1
 2.71621   2.97414   3.28396  281  2
Bartlett's Test (normal distribution)
Test Statistic: 7.096
P-Value       : 0.008
Levene's Test (any continuous distribution)
Test Statistic: 5.341
P-Value       : 0.021

Homogeneity of Variance
Response    diff6090
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper    N  Factor Levels
 2.99524   3.27966   3.62131  281  1
 2.65863   2.91109   3.21433  281  2
Bartlett's Test (normal distribution)
Test Statistic: 3.963
P-Value       : 0.047
Levene's Test (any continuous distribution)
Test Statistic: 0.968
P-Value       : 0.326

Homogeneity of Variance
Response    diff60112
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper    N  Factor Levels
 2.97503   3.25754   3.59688  281  1
 2.57352   2.81790   3.11144  281  2
Bartlett's Test (normal distribution)
Test Statistic: 5.854
P-Value       : 0.016
Levene's Test (any continuous distribution)
Test Statistic: 2.136
P-Value       : 0.144

Homogeneity of Variance

Response    diff60135
Factors     Midpoint
ConfLvl     95.0000
Bonferroni confidence intervals for standard deviations
  Lower     Sigma     Upper    N  Factor Levels
 2.48311   2.71890   3.00212  281  1
 2.31424   2.53400   2.79796  281  2
Bartlett's Test (normal distribution)
Test Statistic: 1.385
P-Value       : 0.239
Levene's Test (any continuous distribution)
```

```
 Test Statistic: 0.953
P-Value       : 0.329

Homogeneity of Variance
Response     diff60157
Factors      Midpoint
ConfLvl      95.0000
Bonferroni confidence intervals for standard deviations
  Lower      Sigma      Upper   N  Factor Levels
 3.32635   3.64222   4.02163  281  1
 2.03992   2.23363   2.46630  281  2
Bartlett's Test (normal distribution)
Test Statistic: 64.321
P-Value       :  0.000
Levene's Test (any continuous distribution)
Test Statistic: 3.376
P-Value       : 0.067
```

p-values:

P-values are often used in hypothesis tests, where you either reject or fail to reject a null hypothesis. The p-value represents the probability of making a Type 1 error, which is rejecting the null hypothesis when it is true. The smaller the p-value, the smaller is the probability that you would be making a mistake by rejecting the null hypothesis. A cut-off value often used is 0.05, that is, reject the null hypothesis when the p-value is less than 0.05. For example, suppose you do a t-test to test the null hypothesis that mu equals 5, versus the alternative hypothesis that it does not equal 5. You would reject the null hypothesis that mu equals 5 if the test yields a very small (for example, less than 0.05) p-value.

APPENDIX G

Correlation Coefficient (Pearson's) Matrix of Vector Variables

Correlations (Pearson) for Vectors using Midpoint Method 2

	avg00	avg022	avg045	avg067	avg090	avg0112	avg0135	avg0157
avg022	0.796							
avg045	0.518	0.687						
avg067	0.340	0.468	0.850					
avg090	0.301	0.380	0.691	0.878				
avg0112	0.428	0.377	0.529	0.668	0.858			
avg0135	0.682	0.462	0.387	0.386	0.502	0.792		
avg0157	0.855	0.614	0.432	0.316	0.334	0.544	0.841	
avg300	0.552	0.557	0.394	0.161	0.178	0.177	0.223	0.431
avg3022	0.434	0.539	0.410	0.175	0.166	0.110	0.082	0.268
avg3045	0.385	0.520	0.513	0.321	0.305	0.195	0.083	0.236
avg3067	0.313	0.446	0.492	0.365	0.382	0.258	0.090	0.204
avg3090	0.314	0.419	0.447	0.328	0.384	0.305	0.142	0.236
avg30112	0.403	0.453	0.415	0.266	0.342	0.322	0.238	0.343
avg30135	0.512	0.503	0.402	0.220	0.277	0.288	0.303	0.462
avg30157	0.573	0.526	0.395	0.195	0.226	0.251	0.322	0.531
avg600	0.367	0.414	0.285	0.078	0.105	0.059	0.045	0.245
avg6022	0.307	0.392	0.277	0.074	0.074	0.004	-0.025	0.170
avg6045	0.295	0.376	0.276	0.086	0.097	0.018	-0.032	0.164
avg6067	0.303	0.376	0.278	0.088	0.110	0.033	-0.018	0.170
avg6090	0.303	0.373	0.276	0.088	0.112	0.042	-0.006	0.183
avg60112	0.329	0.384	0.291	0.096	0.124	0.061	0.020	0.210
avg60135	0.357	0.395	0.293	0.095	0.124	0.077	0.049	0.242
avg60157	0.382	0.406	0.294	0.099	0.134	0.096	0.083	0.284

	avg300	avg3022	avg3045	avg3067	avg3090	avg30112	avg30135	avg30157
avg3022	0.944							
avg3045	0.906	0.948						
avg3067	0.872	0.874	0.954					
avg3090	0.871	0.844	0.912	0.973				
avg30112	0.911	0.850	0.890	0.931	0.969			
avg30135	0.942	0.846	0.867	0.885	0.916	0.968		
avg30157	0.944	0.824	0.834	0.838	0.860	0.920	0.979	
avg600	0.943	0.908	0.900	0.885	0.880	0.899	0.913	0.903
avg6022	0.916	0.914	0.906	0.887	0.866	0.869	0.869	0.854
avg6045	0.920	0.904	0.908	0.906	0.891	0.894	0.891	0.871

avg6067	0.923	0.898	0.910	0.913	0.907	0.913	0.909	0.885
avg6090	0.926	0.889	0.904	0.914	0.916	0.922	0.923	0.900
avg60112	0.934	0.886	0.900	0.908	0.913	0.927	0.931	0.912
avg60135	0.933	0.873	0.888	0.897	0.908	0.930	0.941	0.926
avg60157	0.934	0.859	0.869	0.878	0.892	0.923	0.943	0.937

	avg600	avg6022	avg6045	avg6067	avg6090	avg60112	avg60135
avg6022	0.966						
avg6045	0.967	0.987					
avg6067	0.965	0.969	0.987				
avg6090	0.963	0.954	0.977	0.986			
avg60112	0.968	0.951	0.971	0.984	0.990		
avg60135	0.965	0.940	0.959	0.974	0.982	0.989	
avg60157	0.966	0.921	0.941	0.957	0.966	0.976	0.985

APPENDIX H

Tests of Assumption of Regression Analysis

Regression Analysis
Size 2

The regression equation is
avg3090 = - 34.5 + 0.145 B-C Arc + 0.435 Breadth (mm)

Predictor	Coef	StDev	T	P
Constant	-34.46	31.77	-1.08	0.283
B-C Arc	0.14549	0.03413	4.26	0.000
Breadth	0.4346	0.1988	2.19	0.034

S = 2.917 R-Sq = 33.9% R-Sq(adj) = 31.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	218.27	109.14	12.83	0.000
Error	50	425.35	8.51		
Total	52	643.63			

Source	DF	Seq SS
B-C Arc	1	177.62
Breadth	1	40.66

Unusual Observations

Obs	B-C Arc	avg3090	Fit	StDev Fit	Residual	St Resid
21	384	82.550	88.273	1.097	-5.723	-2.12R
30	348	89.450	83.063	0.554	6.387	2.23R
42	360	79.900	86.125	0.513	-6.225	-2.17R
53	358	80.450	86.271	0.647	-5.821	-2.05R

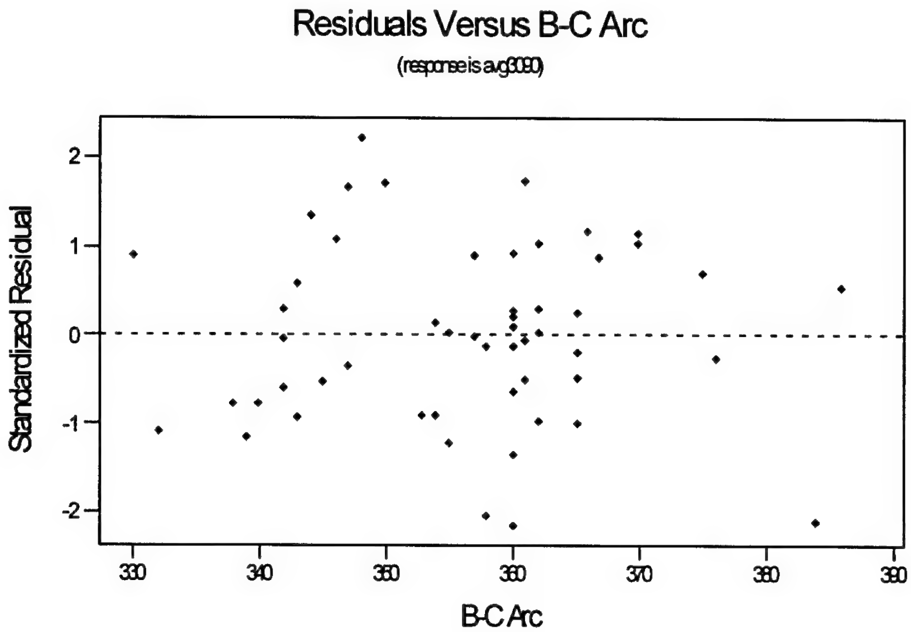
R denotes an observation with a large standardized residual

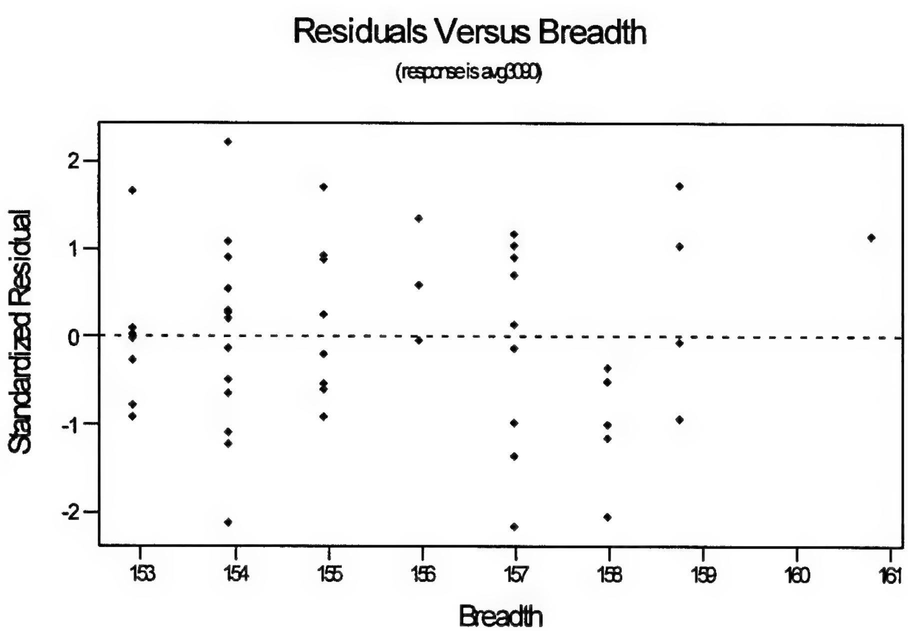
Fit	StDev Fit	95.0% CI	95.0% PI	
91.549	1.433	(88.670, 94.428)	(85.021, 98.078)	X

X denotes a row with X values away from the center

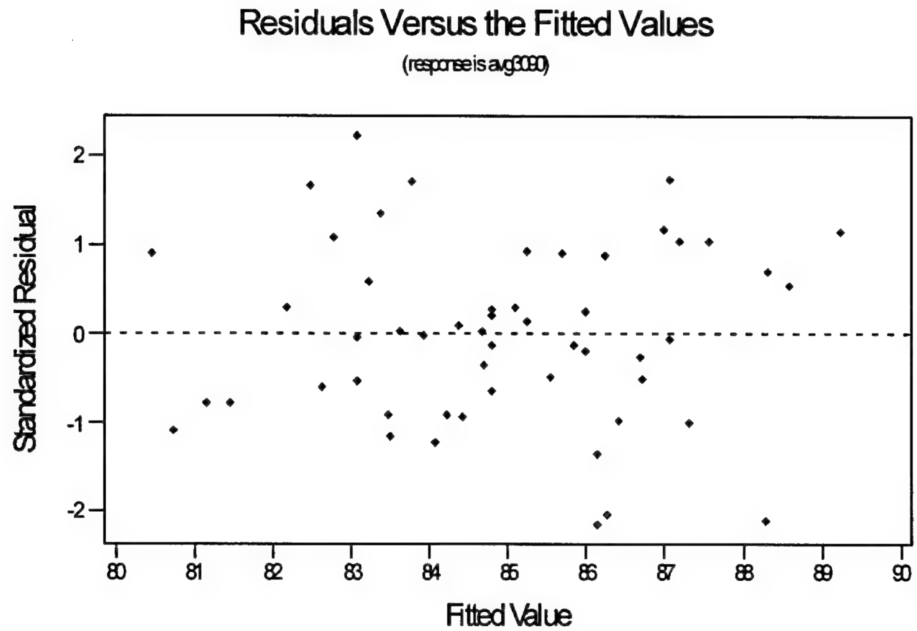
No evidence of lack of fit (P > 0.1)

Tests for Assumption of Linearity: Since the residuals do not appear functionally related to the predictor values, (that is, head length (mm) and head breadth (mm)), , the assumption of linearity can not be rejected.

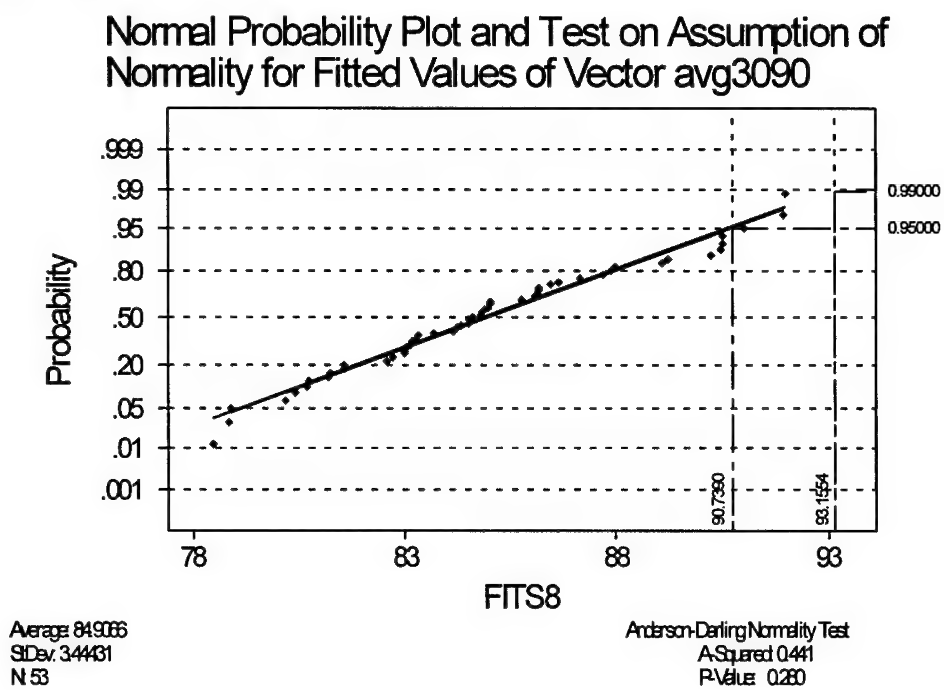




Test for the Assumption of Homoscedasticity: Since no obvious patterns could be identified from the plot of residuals versus fitted values, the assumption of homoscedasticity can not be rejected.



Test for the Assumption of Normality: The null hypothesis is that each fitted value is normally distributed. The alternative hypothesis is that each fitted value is not normally distributed. Since our p-value is greater than than our cut-value 0.05, we can not reject the null hypothesis.



Regression Analysis

The regression equation is
avg3090 = 82.9 - 0.908 PC1 - 0.616 PC3 + 0.613 PC4

Predictor	Coef	StDev	T	P
Constant	82.9480	0.1453	570.89	0.000
PC1	-0.90846	0.03081	-29.49	0.000
PC3	-0.6155	0.1181	-5.21	0.000
PC4	0.6128	0.1107	5.53	0.000

S = 0.7386   R-Sq = 95.8%   R-Sq(adj) = 95.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	616.89	205.63	376.89	0.000
Error	49	26.73	0.55		
Total	52	643.63			

Source	DF	Seq SS
PC1	1	594.54
PC3	1	5.64
PC4	1	16.71

Unusual Observations

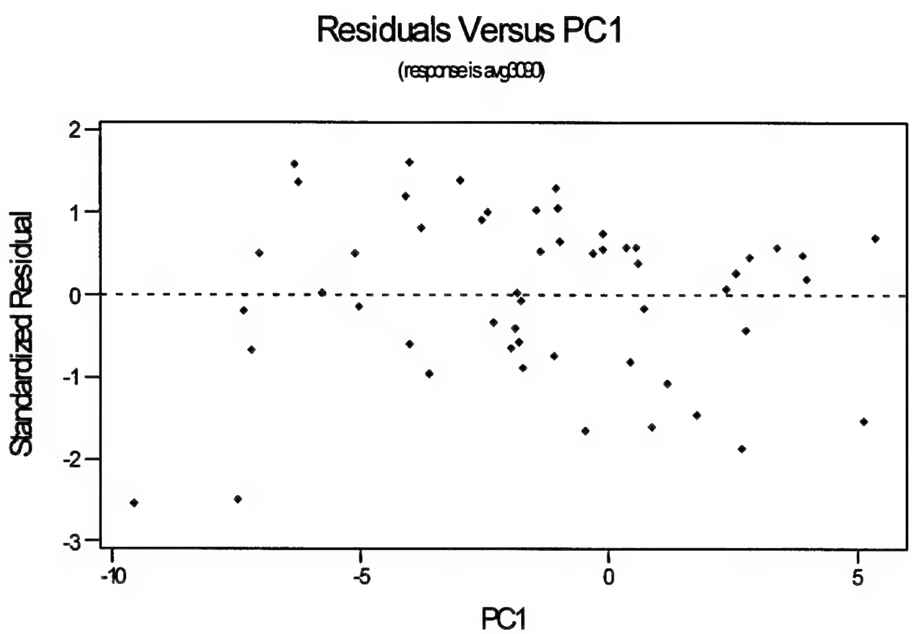
Obs	PC1	avg3090	Fit	StDev Fit	Residual	St Resid
9	-9.56	90.300	92.021	0.289	-1.721	-2.53R
35	-7.49	88.700	90.467	0.211	-1.767	-2.50R

R denotes an observation with a large standardized residual

Fit	StDev Fit	99.0% CI	99.0% PI	
95.191	0.456	(93.969, 96.413)	(92.865, 97.518)	XX

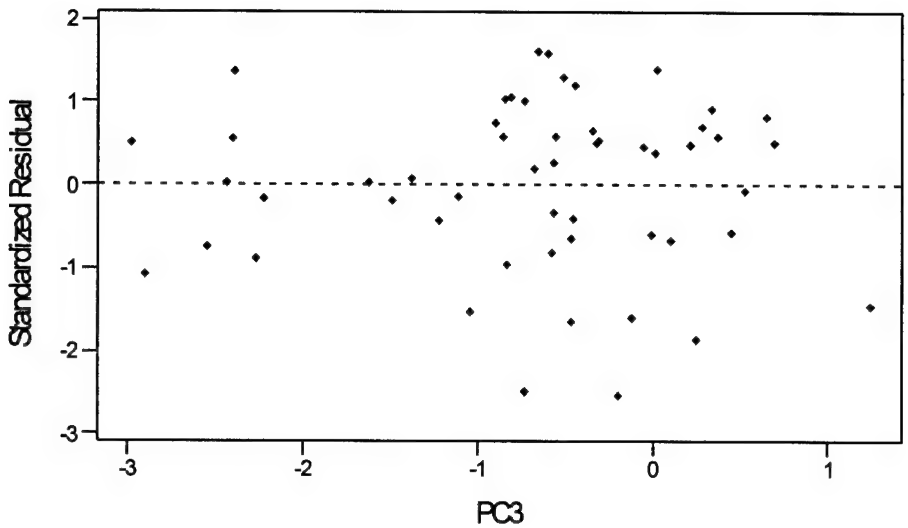
X denotes a row with X values away from the center
XX denotes a row with very extreme X values

No replicates. Cannot do pure error test.


Residuals Versus PC1
(response is avg3090)
Standardized Residual
2
1
0
-1
-2
-3
-10
-5
0
5
PC1

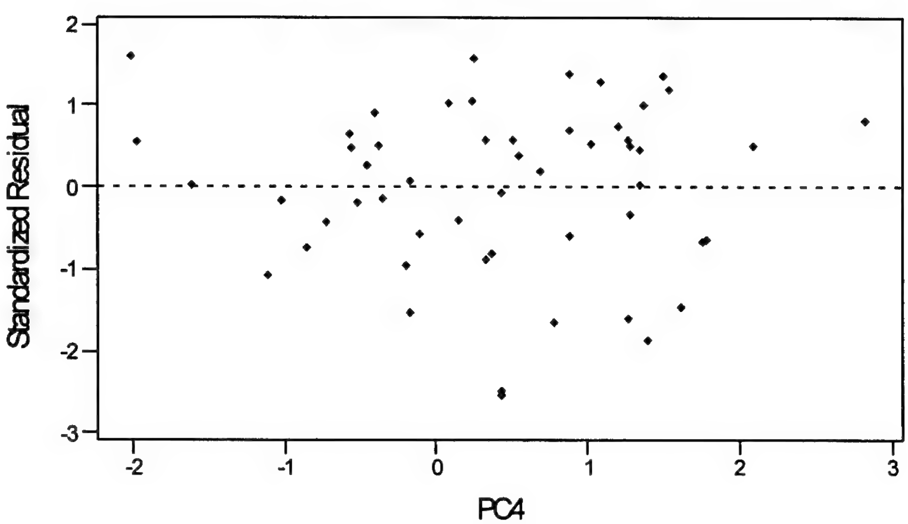
Residuals Versus PC3

(response is avg3090)



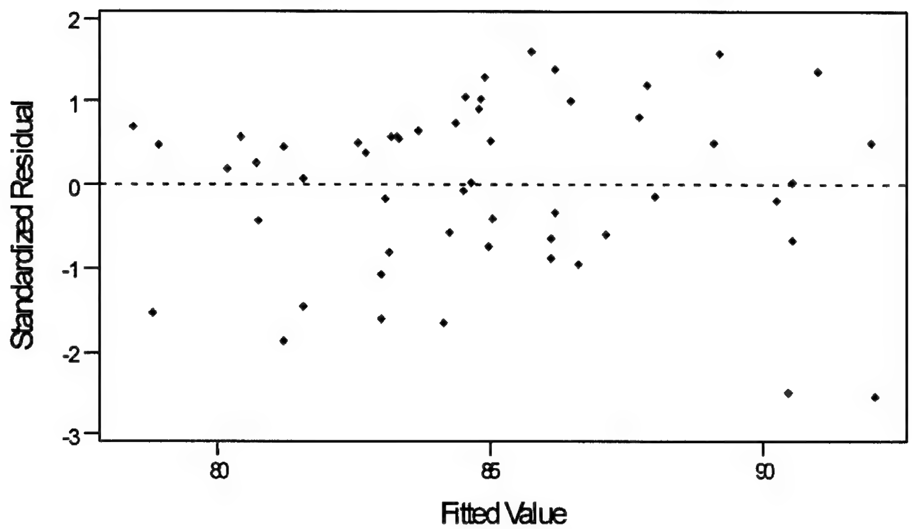
Residuals Versus PC4

(response is avg3090)

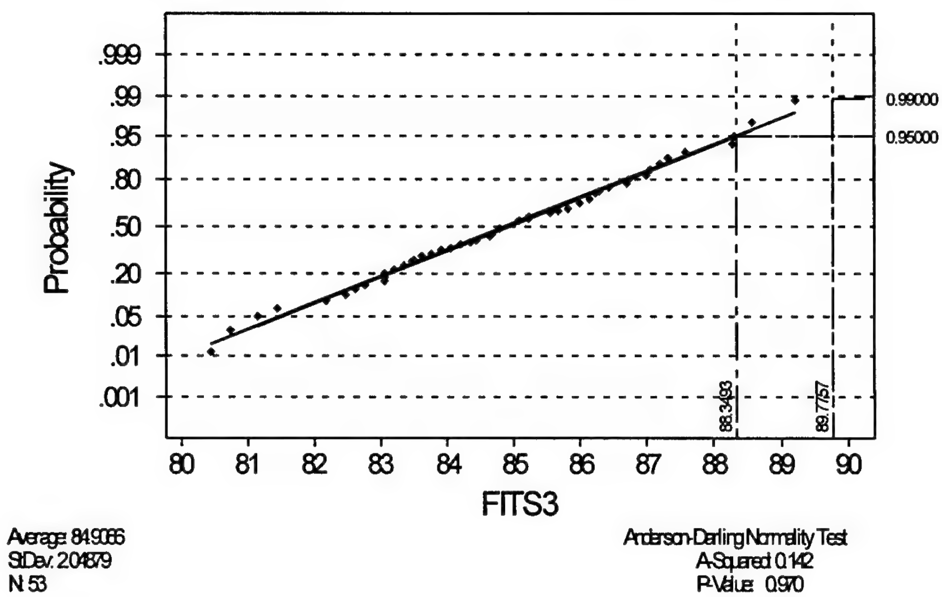


Residuals Versus the Fitted Values

(response is avg3090)



Normal Probability Plot and Test for Assumption of Normality for Fitted Values of Vector Avg3090



APPENDIX I

Subject's Principal Component Scores for First Four Eigenvectors for the Sample of Aviators from 1990 U.S. Air Force Anthropometric Survey of Aviators

Subject ID Number	PC1	PC2	PC3	PC4
1	-0.9356	1.36561	2.26386	0.15816
2	-5.3298	-2.46831	1.97575	-0.90079
3	-4.003	-1.76893	-0.00678	0.88009
6	-0.6885	-0.48679	0.72627	0.91506
28	-1.9694	-0.99123	-0.46198	1.77422
32	-4.0142	2.27721	-0.65372	-2.01018
33	-2.8679	-5.79432	0.24952	0.05996
34	-5.2457	0.86417	0.64471	-0.09918
39	-4.7524	-0.46329	0.68168	-0.1237
43	0.4978	-0.79386	-0.1505	0.43399
45	-1.4816	-2.3311	-1.09574	-0.27067
46	-4.7709	2.27107	-0.01682	0.88624
48	-5.7046	0.66539	-0.85296	0.50939
49	-4.9333	-2.02243	1.75992	-0.73026
50	3.9217	1.76655	0.21501	-0.56409
51	-4.0007	-2.44163	2.27915	-0.99298
52	-0.4425	-5.45921	-0.2155	-0.92551
53	-3.3829	-5.68881	1.30344	-1.04428
54	-3.9594	-0.31826	0.28753	2.34494
55	2.4794	-1.03352	-0.47216	-0.2192
56	-0.1451	-0.42921	-0.07964	-1.56341
57	1.5888	-0.13372	1.50561	0.07907
58	-7.9565	-1.63831	3.28782	-0.59269
59	4.3426	2.47513	-0.5181	-0.43223
60	-4.492	-5.36532	1.14072	0.04742
61	-2.7327	0.46302	0.49777	-1.09474
62	-5.4127	-3.18646	-0.5716	0.09324
63	0.2665	1.5569	0.56707	-0.52989
64	-6.7955	1.32828	1.02131	0.50379

65	-5.6462	2.22543	0.05695	-0.29439
66	-4.8455	-1.99709	0.00386	-0.19793
68	0.2455	-3.69899	-0.70958	0.70784
69	-9.5557	0.57403	-0.20425	0.43385
70	1.5388	-0.64722	-0.19904	-0.06381
71	-2.6027	-0.12557	0.30233	-1.94723
72	-5.7011	0.7553	-0.55865	-0.43174
73	-1.0166	-0.04211	-1.10583	-0.02902
74	3.3779	0.20685	0.36321	1.26618
75	-5.1381	-2.43957	-1.06819	-0.20436
79	-0.3599	-1.80812	-1.23275	0.6436
80	-2.3472	-0.10151	0.5335	-1.45932
82	0.6911	-2.40715	-0.01961	0.38668
83	-5.2154	-4.7889	0.4855	-0.38016
84	-8.2268	-3.6321	0.46919	1.73883
85	-5.0629	-3.03389	0.18708	-1.49261
86	-3.8378	-1.06645	1.59239	-0.38149
95	1.7714	0.7215	1.23341	1.6097
97	0.9583	1.72415	1.03738	0.34676
98	-2.4953	2.14773	2.05099	-0.77663
99	3.4908	2.77233	1.58072	-0.24631
100	-1.1375	0.32389	-0.51612	-0.89246
112	-2.9305	3.37988	-1.71981	-0.94075
113	0.929	-0.13133	-0.25507	-0.72457
114	-4.8115	3.1714	-3.04277	1.00595
115	2.2698	2.26622	1.29748	1.20157
116	1.072	0.21278	-0.41251	0.80259
117	1.2032	1.02464	-2.15372	0.61699
118	-2.9409	2.1374	0.16206	0.57039
119	-0.5353	4.50353	0.41295	0.03166
120	2.4327	-0.75732	0.66026	0.37571
121	-5.7956	-3.28335	-2.44191	1.34075
122	-1.7799	0.99361	0.51948	0.42595
124	-8.8283	-1.98066	-1.04275	0.46111
125	-4.5537	-0.45429	-0.12423	-0.39378
126	-4.2175	-0.12333	0.61624	-0.85218
127	0.6037	-2.94396	1.93826	-0.07618
128	1.5623	1.9217	-1.80544	-0.031
129	1.7143	0.12499	1.03683	0.43244
130	-5.1499	-1.34486	0.07179	-1.88985
131	-1.8357	1.68809	-1.62951	-1.60997
132	-6.3213	-2.65746	-0.60564	0.24808
133	-2.3215	-0.52076	-0.57126	1.28331
134	7.9471	-0.74421	-0.88645	-0.93628
135	0.3232	-0.4174	1.11837	-1.89224
136	-5.0516	-0.19206	-1.11464	-0.35909
137	2.7541	-0.65425	0.96908	-0.55148
138	2.3559	1.06564	-1.38357	-0.17503
139	-6.9102	0.7563	0.63389	-1.33218

140	-2.634	-0.70678	-0.23985	1.36029
141	0.1218	-0.76366	0.75949	0.24341
142	-7.359	2.66673	-1.48761	-0.51726
143	-6.2445	-0.63165	-2.39341	1.49024
144	-0.6382	-4.75792	-0.80865	-0.65467
145	1.1753	1.22266	-2.90551	-1.11212
146	4.5655	-0.34181	-2.24339	-0.0456
147	-1.0088	0.99724	1.24551	1.30447
149	-3.7946	2.23059	0.64274	2.81905
150	-3.5607	3.71864	0.22431	-1.00775
151	-8.7752	2.61106	1.87877	-0.948
152	0.3964	-3.38718	-1.21458	0.49886
153	3.4659	2.37435	1.13367	-0.29681
154	4.5132	0.75042	1.95883	1.57335
155	1.0198	-1.04683	-2.74975	1.91988
156	0.9588	0.54402	1.15919	-1.03672
157	0.4325	3.46469	-0.58199	0.36597
158	-4.4139	0.46076	0.90842	-0.73693
159	0.6937	0.93816	-2.22483	-1.02594
160	-1.9027	2.01569	-0.45354	0.1479
162	-1.7472	4.3875	-2.2675	0.32726
163	1.0987	-0.25026	0.64486	0.44671
164	-1.0243	2.94412	-0.81691	0.23956
165	-0.1073	1.44401	-2.40202	-1.97792
166	1.3372	3.47553	-0.78084	1.16354
167	2.6024	-0.74389	0.41977	-0.59412
168	3.0183	1.2023	-0.30698	-1.06971
169	-1.0682	1.517	-0.51017	1.09002
170	-4.1897	2.68347	2.00996	-0.94769
171	12.8984	-1.1084	1.44182	-0.1188
172	0.1231	2.10421	-2.10042	-0.50517
173	0.2777	1.79997	0.6648	-1.47067
174	1.9147	1.95813	-0.33211	1.30913
175	1.8223	0.27109	1.06248	-1.00279
176	-2.0476	0.60849	2.10679	0.89716
177	1.068	-0.67081	1.5571	1.20246
178	-4.1243	-1.8065	0.35974	0.38083
179	3.0537	2.19306	1.77524	0.60226
182	-2.3306	0.3348	-0.87896	0.31839
183	-4.5258	0.68912	0.89203	-0.51231
184	-4.7218	2.37219	0.77716	-0.3159
185	-1.6419	-0.77491	0.78158	1.1075
186	-3.2126	-0.15704	-1.47171	-1.0765
187	0.5171	1.96523	0.99316	0.72084
188	1.9757	1.76564	0.63518	-0.44127
189	-0.975	0.11269	2.08972	-0.07469
190	5.1368	2.00624	-1.05005	-0.17391
191	2.8464	1.56454	-0.05106	1.34013
192	2.9053	-1.33044	0.26693	0.37007

194	-0.2838	4.21354	-0.96732	0.56943
195	0.7761	1.34063	-1.32842	-0.692
196	1.3157	-0.46151	-1.26307	1.34018
197	-1.4917	-1.59635	1.78493	1.22484
198	-1.3705	0.49386	-0.31517	1.0233
200	-5.1378	0.58647	-0.3221	2.08186
201	-0.1333	1.02646	-0.89676	1.19669
202	1.7543	1.48378	2.5987	1.8162
203	0.8457	3.15996	-0.64091	0.7184
204	0.2474	-1.03241	-1.45091	-0.75794
205	3.7942	2.10496	1.2991	1.67297
206	6.2656	-1.8046	-0.52188	1.67919
207	1.1132	-0.71085	0.99148	-0.00596
208	-0.086	-1.27336	-1.43501	-1.33319
211	4.9068	-0.58987	-3.27601	0.10002
212	5.8376	-2.62764	-1.56356	0.51931
213	4.8582	-3.66244	-0.37876	-0.52075
214	-2.6627	1.23235	-0.47373	0.55722
217	-7.2221	-1.16363	0.09878	1.74954
218	2.8468	6.35888	-2.34918	1.28138
219	1.4781	-0.21907	-1.38439	0.40036
220	-0.1162	0.0497	0.42214	1.14841
221	0.9396	-4.38606	0.8006	-0.10665
222	-8.2661	0.58784	0.63528	-0.7938
223	-5.1174	-0.29959	1.27362	-1.10584
224	1.0936	-1.98392	-0.28722	0.27546
225	-2.4068	0.32543	-2.19927	1.00185
226	-0.071	1.47106	-2.52767	0.71358
227	-1.7151	-3.03304	0.66234	-0.32623
228	0.3382	2.4768	-0.55796	0.32342
229	-4.0496	-0.7668	-3.74925	-0.23809
230	0.1917	3.85504	2.43414	-0.48681
232	-1.0878	3.04851	-2.54588	-0.86082
233	3.0184	1.17764	-2.57331	-0.46954
234	0.6729	-0.72835	1.45975	-0.14289
235	2.4693	0.33033	-0.8774	0.31719
236	-7.4881	1.19557	-0.73458	0.43054
237	-0.6727	-0.30876	-2.3003	-0.52877
238	4.5299	-1.12771	-2.26329	0.77139
240	2.2107	2.55077	2.50419	-0.64433
241	0.4672	-0.26871	1.19153	-1.1989
242	-1.4634	1.60995	-0.84429	0.07905
243	-2.7434	-4.44789	-1.2396	0.2257
244	2.4069	1.98374	-0.5136	-1.00577
245	0.5594	1.77409	-0.85759	0.51003
246	-2.7325	-3.5426	-0.8923	0.99495
247	4.1394	0.40083	0.84025	-0.20781
248	2.808	-2.32272	1.00432	-0.49193
249	7.2426	2.34933	-0.1785	0.35282

250	-2.5473	2.47776	0.33803	-0.40919
251	-1.7303	5.28885	-0.87152	-1.14631
252	5.5272	-1.49483	1.0092	-0.46074
253	3.8649	-5.67047	-2.54633	0.47654
254	-0.2026	0.02308	3.17419	-0.46765
255	4.9527	-0.74242	0.40892	-0.74399
257	-2.7661	-1.79285	-0.52883	-0.11971
258	1.1646	-1.88209	0.44496	1.3617
259	-0.5666	0.33299	-0.04502	-1.40374
260	-0.0457	0.23514	-1.07554	-1.92476
261	1.5277	-2.71631	0.15281	0.97345
263	0.2282	2.44102	1.49344	-0.53872
264	5.9138	-2.29709	0.63286	-1.29286
265	-5.6187	-5.31462	0.85944	0.46901
266	2.8484	0.61268	1.33687	0.14656
268	6.8476	0.56046	0.63995	-0.49617
269	1.4507	1.00221	-1.48235	-1.44363
270	1.9729	0.15775	-0.34049	1.12126
271	3.1154	2.04628	0.82584	-1.21579
272	-3.2486	0.15268	-0.91672	0.54841
273	3.7724	0.96015	1.20975	-0.47455
275	7.4438	-1.76516	-1.40094	-0.18312
276	3.5936	-0.8194	0.05716	-0.27166
277	-2.4574	1.88784	-0.73352	1.37241
278	-3.6706	-2.42602	-1.27021	-1.17765
279	-1.7283	1.34862	-0.39224	0.30497
280	-3.7025	1.45903	1.85786	0.57523
281	4.2369	1.63471	2.77586	0.06548
282	-5.2966	1.90427	2.79493	-0.20586
283	1.7226	0.03401	1.23623	0.831
284	-0.3188	1.47927	0.22449	0.45407
285	6.4379	1.59143	1.02225	-0.53206
286	6.4311	-1.65781	0.31617	-0.00354
287	2.8407	-1.79341	-0.30822	1.61261
288	0.066	0.49774	-0.2024	-0.19572
289	6.9763	0.58645	1.65878	-0.31651
290	7.0346	0.03454	-0.37364	-0.33051
291	1.6993	-2.65601	0.53441	0.45273
292	0.8993	3.53652	0.19776	1.1239
293	-1.1728	-0.73892	-1.73497	-0.38305
294	-2.4415	-2.03729	2.28699	1.19105
295	8.341	-1.91726	0.66473	0.89243
296	-0.4889	1.04642	-0.4635	0.77924
297	4.7315	-0.69471	0.30308	0.50519
298	1.9087	1.00225	-0.62931	0.37356
299	5.8306	1.61396	-0.46629	-0.71707
300	-0.7736	-2.6108	1.84907	0.03579
301	13.1378	-0.67632	0.61004	-0.26506
302	-2.9827	0.68178	0.02114	0.88512

305	2.6732	1.20278	0.24563	1.39729
307	1.9324	-1.6708	-0.09762	-0.02688
308	3.8977	-0.8263	1.47598	-1.5915
309	1.2475	-2.5226	-1.02403	1.14162
310	1.7734	-1.3251	3.17001	-0.56082
311	-3.3644	-2.46207	-0.76363	-0.77923
312	6.5473	0.62686	-1.40119	0.70376
314	-1.5788	2.20654	1.48708	-0.25181
315	4.9888	-3.08943	1.23973	-0.65991
316	0.8672	1.10149	-0.12294	1.2629
317	3.7792	-2.64039	-0.41494	0.03705
318	1.0468	-1.61897	0.37171	0.24634
319	5.5306	2.4825	1.28539	0.1896
320	1.4112	1.98722	0.45481	-1.53317
321	-0.9629	-0.10468	-0.34245	-0.57336
322	4.2809	-2.54486	-1.63828	0.2905
323	0.3684	2.12282	0.31208	-0.63158
324	-0.3313	0.68465	0.69712	-0.38238
325	3.9888	0.43186	-0.67267	0.68769
326	-7.0388	2.5006	-2.97958	1.27742
327	7.9402	-0.64787	1.55344	-0.2756
328	-7.3133	2.8231	2.17574	-0.1722
329	5.3287	-4.25241	1.40514	-0.34533
330	4.8756	0.23601	0.79259	-0.31534
331	-2.3144	-0.98312	-0.45188	-1.083
332	2.8715	0.41605	-0.44209	-0.20262
333	-6.0691	0.45029	1.31682	0.02526
334	1.5641	-1.40786	-1.80815	-0.47827
335	-4.0875	-0.15817	-0.44013	1.53557
336	-6.7905	0.057	1.69205	1.40295
337	4.2381	0.32831	1.99381	0.98562
338	5.4485	-3.57499	0.7245	0.61576
339	5.346	0.21059	0.27572	0.87749
340	2.6127	-2.84653	-1.86117	0.58565
341	-3.6215	-0.15364	-0.32706	0.34126
342	11.5241	-2.27486	-0.88301	-1.05646
343	0.606	-2.22759	0.86711	-1.20529
344	-2.7991	0.68544	1.15386	-0.94612
345	-3.633	-0.11153	-0.83293	-0.20314
346	8.6094	-1.15924	-0.34995	-0.88786
347	3.5802	-2.74992	-0.45894	0.48822
348	-0.7761	1.45921	0.68282	0.83096
349	2.8431	3.29406	-0.29227	-0.22635
350	2.1186	5.08903	-0.27172	-2.01546
351	3.2981	-2.0179	-0.26414	0.4818
352	2.6977	-0.83253	-1.08082	-1.06305
353	2.5464	2.74335	-0.57069	-0.46433
354	1.9442	-0.96841	-1.21164	1.65043
355	-1.8255	-1.62637	0.44581	-0.11369

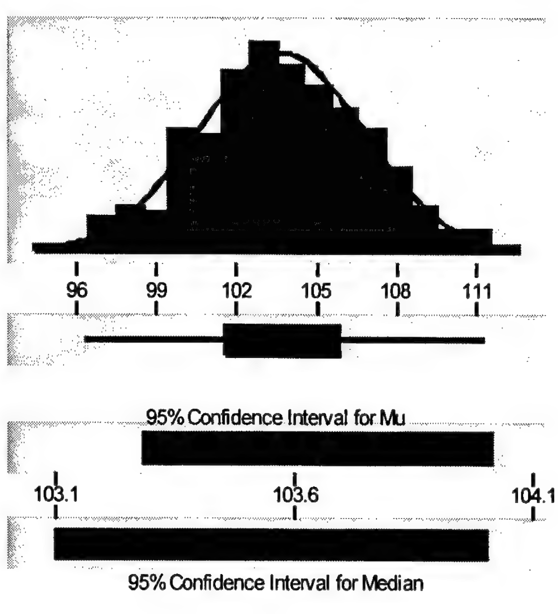
356	0.6082	0.56851	0.00968	0.54166
357	2.755	0.69125	-1.22019	-0.72411
358	2.4855	-2.70474	-1.80655	-1.0681
359	-1.081	-0.12225	-1.01263	0.26112
360	-5.0998	-0.41941	1.09432	-1.45731
361	4.6461	1.00561	0.12411	-0.93894
362	4.0266	3.11667	0.98178	-0.0406
363	-4.1901	-1.51769	-3.32317	-5.67709
365	1.4216	1.44624	0.93163	-0.43202

APPENDIX J

Normality Tests and Descriptive Statistics for the Vectors

Interpreting the p-values: The Anderson-Darling test listed below is a test for normality based on an empirical cumulative distribution function. The null hypothesis of the test is that the data are normal; the alternative hypothesis is that the data are not normal. A p-value less than the selected cut-value of 0.05, means that it is necessary to reject the null hypothesis, that is, to reject the hypothesis that the data are normal. Descriptive statistics including histograms and normal curves are provided below for each of the vectors.

Descriptive Statistics



Variable: v00

Anderson-Darling Normality Test

A-Squared:	0.116
P-Value:	0.991
Mean	103.647
StDev	3.117
Variance	9.71372
Skewness	1.15E-02
Kurtosis	-2.1E-01
N	281
Minimum	95.100
1st Quartile	101.550
Median	103.600
3rd Quartile	105.800
Maximum	112.400

95% Confidence Interval for Mu

103.281	104.013

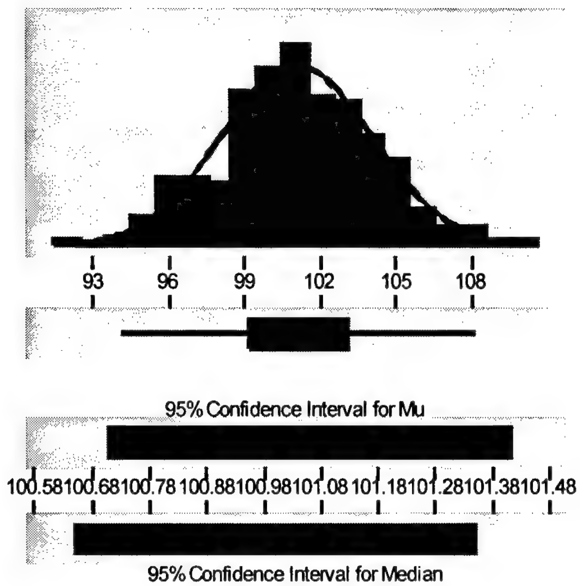
95% Confidence Interval for Sigma

2.879	3.398

95% Confidence Interval for Median

103.100	104.000

Descriptive Statistics



Variable: avg022

Anderson-Darling Normality Test

A-Squared:	0.214
P-Value:	0.850
Mean	101.061
StDev	3.011
Variance	9.06704
Skewness	1.49E-02
Kurtosis	-6.0E-02
N	281
Minimum	92.400
1st Quartile	99.200
Median	101.050
3rd Quartile	103.075
Maximum	109.800

95% Confidence Interval for Mu

100.707 101.415

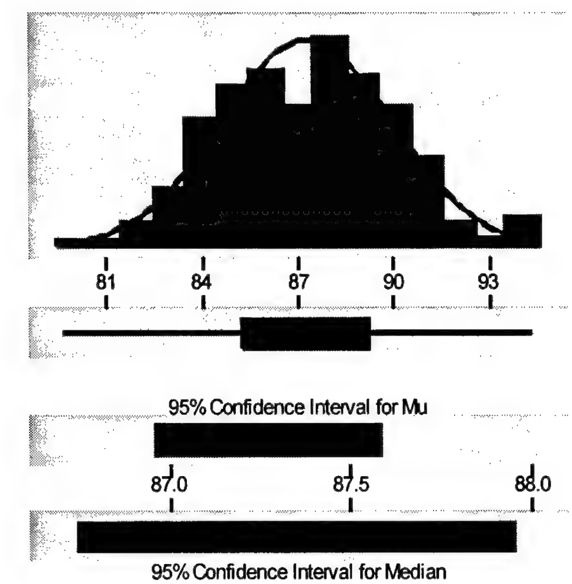
95% Confidence Interval for Sigma

2.781 3.283

95% Confidence Interval for Median

100.650 101.350

Descriptive Statistics



Variable: avg045

Anderson-Darling Normality Test

A-Squared:	0.650
P-Value:	0.089
Mean	87.2696
StDev	2.6555
Variance	7.05142
Skewness	9.46E-02
Kurtosis	-2.8E-01
N	281
Minimum	79.6500
1st Quartile	85.2000
Median	87.4000
3rd Quartile	89.1250
Maximum	94.2500

95% Confidence Interval for Mu

86.9577 87.5814

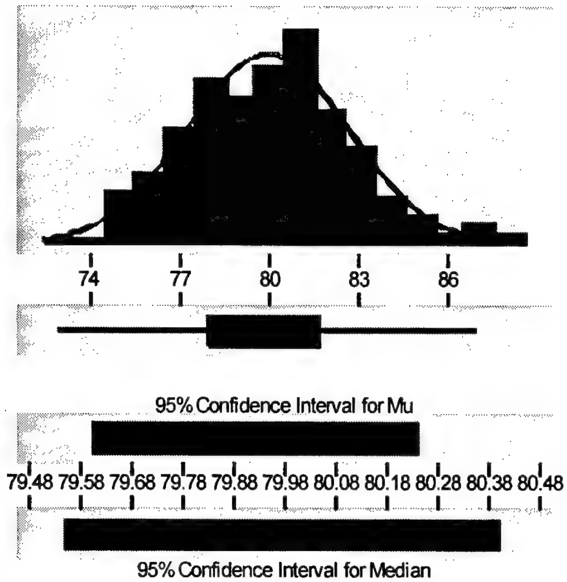
95% Confidence Interval for Sigma

2.4526 2.8952

95% Confidence Interval for Median

86.7500 87.9500

Descriptive Statistics



Variable: avg067

Anderson-Darling Normality Test

A-Squared:	0.453
P-Value:	0.269
Mean	79.9222
StDev	2.6992
Variance	7.28582
Skewness	0.252214
Kurtosis	1.52E-02
N	281
Minimum	72.9000
1st Quartile	77.9000
Median	79.9000
3rd Quartile	81.5750
Maximum	87.9500

95% Confidence Interval for Mu

79.6053 80.2392

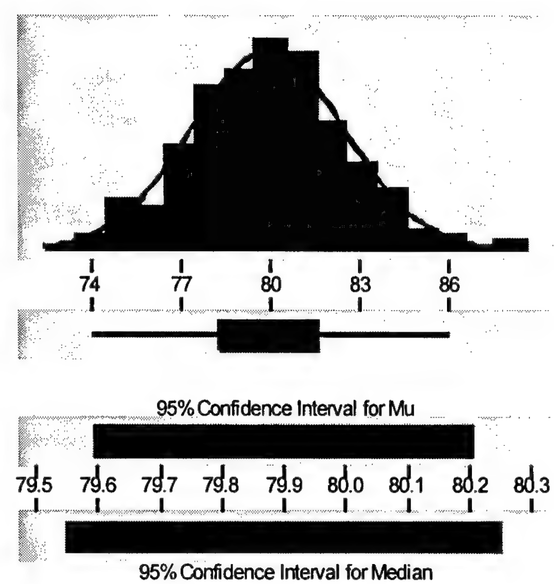
95% Confidence Interval for Sigma

2.4930 2.9430

95% Confidence Interval for Median

79.5500 80.4000

Descriptive Statistics



Variable: avg090

Anderson-Darling Normality Test

A-Squared:	0.292
P-Value:	0.603
Mean	79.8995
StDev	2.5894
Variance	6.70504
Skewness	0.154898
Kurtosis	0.263091
N	281
Minimum	73.1000
1st Quartile	78.2500
Median	80.0500
3rd Quartile	81.5000
Maximum	88.4000

95% Confidence Interval for Mu

79.5954 80.2035

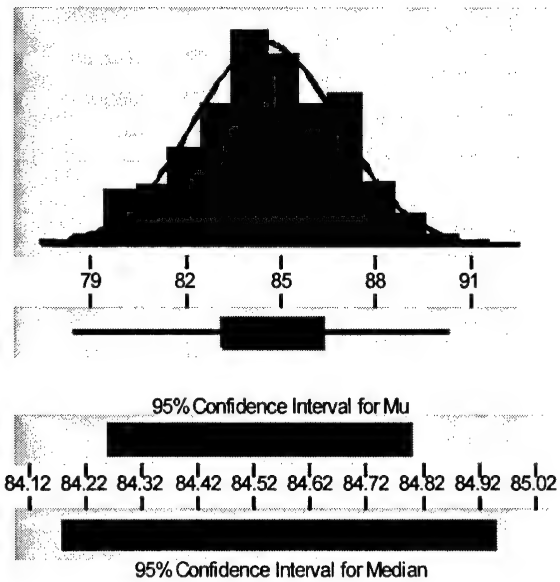
95% Confidence Interval for Sigma

2.3916 2.8232

95% Confidence Interval for Median

79.5500 80.2500

Descriptive Statistics



Variable: avg0112

Anderson-Darling Normality Test

A-Squared:	0.403
P-Value:	0.354
Mean	84.5294
StDev	2.2867
Variance	5.22909
Skewness	-9.3E-02
Kurtosis	-1.9E-01
N	281
Minimum	78.4500
1st Quartile	83.1500
Median	84.5500
3rd Quartile	86.3000
Maximum	91.3500

95% Confidence Interval for Mu

84.2608 84.7979

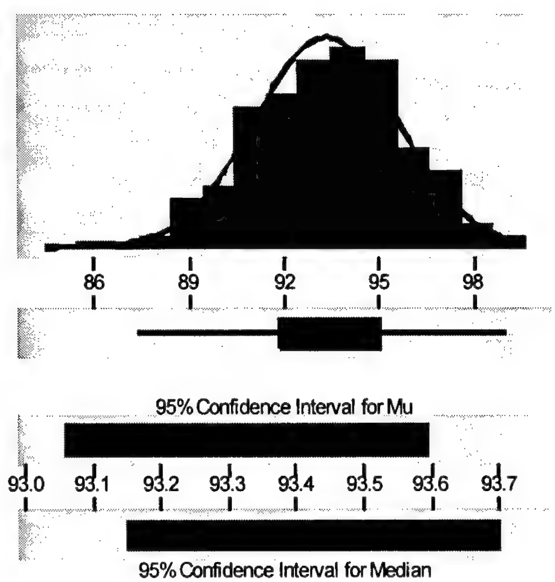
95% Confidence Interval for Sigma

2.1120 2.4932

95% Confidence Interval for Median

84.1801 84.9500

Descriptive Statistics



Variable: avg0135

Anderson-Darling Normality Test

A-Squared:	0.298
P-Value:	0.585
Mean	93.3269
StDev	2.2736
Variance	5.16948
Skewness	-1.8E-01
Kurtosis	-1.7E-01
N	281
Minimum	85.9500
1st Quartile	91.8500
Median	93.4000
3rd Quartile	94.9500
Maximum	98.9500

95% Confidence Interval for Mu

93.0599 93.5939

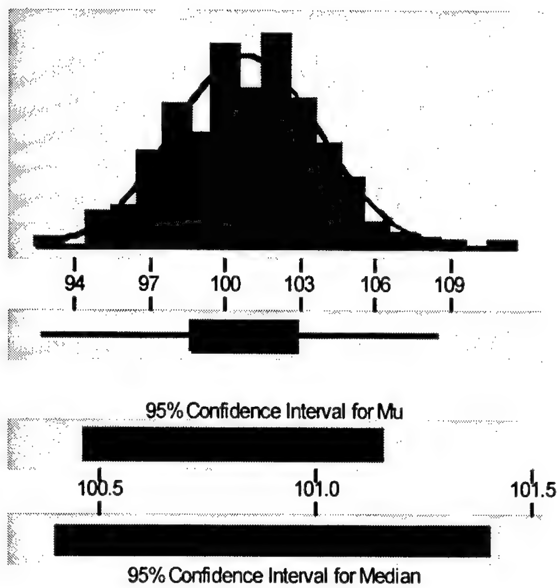
95% Confidence Interval for Sigma

2.0999 2.4790

95% Confidence Interval for Median

93.1500 93.7000

Descriptive Statistics



Variable: avg0157

Anderson-Darling Normality Test

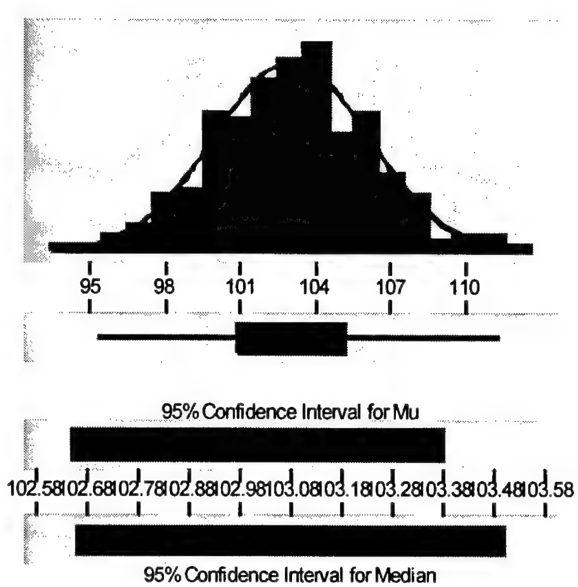
A-Squared:	0.305
P-Value:	0.567
Mean	100.808
StDev	2.910
Variance	8.46987
Skewness	4.94E-02
Kurtosis	0.123517
N	281
Minimum	92.650
1st Quartile	98.650
Median	100.950
3rd Quartile	102.775
Maximum	111.300

95% Confidence Interval for Mu
100.467 101.150

95% Confidence Interval for Sigma
2.688 3.173

95% Confidence Interval for Median
100.400 101.400

Descriptive Statistics



Variable: v0180

Anderson-Darling Normality Test

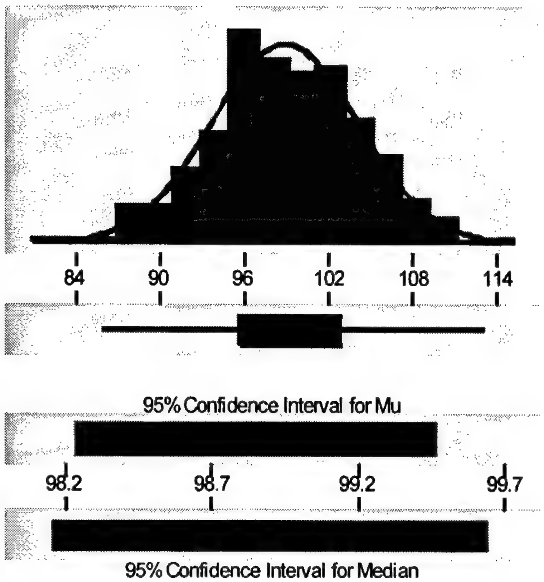
A-Squared:	0.147
P-Value:	0.966
Mean	103.015
StDev	3.092
Variance	9.55911
Skewness	3.50E-02
Kurtosis	-7.2E-03
N	281
Minimum	94.200
1st Quartile	100.900
Median	103.000
3rd Quartile	105.150
Maximum	112.200

95% Confidence Interval for Mu
102.652 103.378

95% Confidence Interval for Sigma
2.856 3.371

95% Confidence Interval for Median
102.660 103.500

Descriptive Statistics



Variable: v300

Anderson-Darling Normality Test

A-Squared:	0.277
P-Value:	0.653
Mean	98.8534
StDev	5.2306
Variance	27.3596
Skewness	-1.7E-01
Kurtosis	-6.4E-02
N	281
Minimum	82.400
1st Quartile	95.550
Median	98.800
3rd Quartile	102.650
Maximum	113.000

95% Confidence Interval for Mu

98.239 99.468

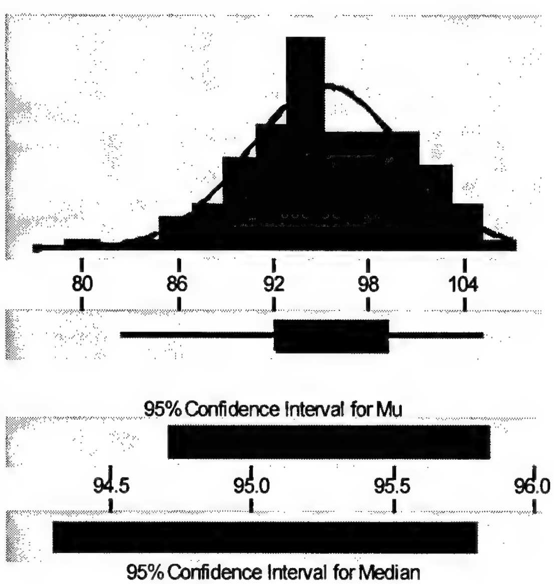
95% Confidence Interval for Sigma

4.831 5.703

95% Confidence Interval for Median

98.160 99.640

Descriptive Statistics



Variable: avg3022

Anderson-Darling Normality Test

A-Squared:	0.586
P-Value:	0.126
Mean	95.2728
StDev	4.8284
Variance	23.3139
Skewness	-2.1E-01
Kurtosis	-9.6E-02
N	281
Minimum	79.200
1st Quartile	92.225
Median	94.800
3rd Quartile	98.975
Maximum	105.200

95% Confidence Interval for Mu

94.706 95.840

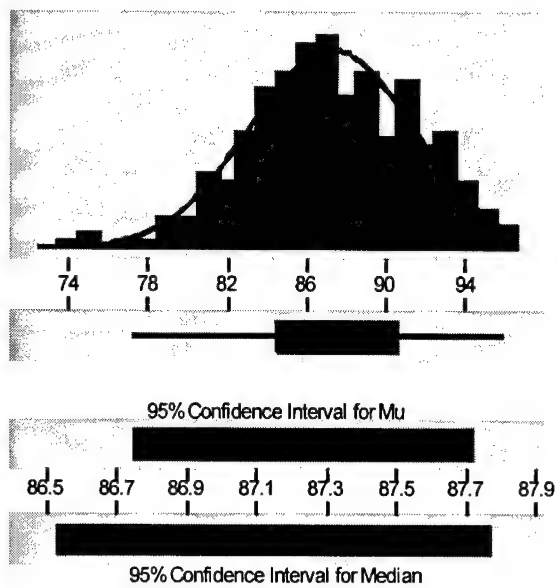
95% Confidence Interval for Sigma

4.460 5.264

95% Confidence Interval for Median

94.300 95.790

Descriptive Statistics



Variable: avg3045

Anderson-Darling Normality Test

A-Squared:	0.451
P-Value:	0.272
Mean	87.2338
StDev	4.1302
Variance	17.0582
Skewness	-2.5E-01
Kurtosis	-9.3E-02
N	281
Minimum	73.6000
1st Quartile	84.4750
Median	87.0500
3rd Quartile	90.4750
Maximum	95.9500

95% Confidence Interval for Mu

86.7488 87.7188

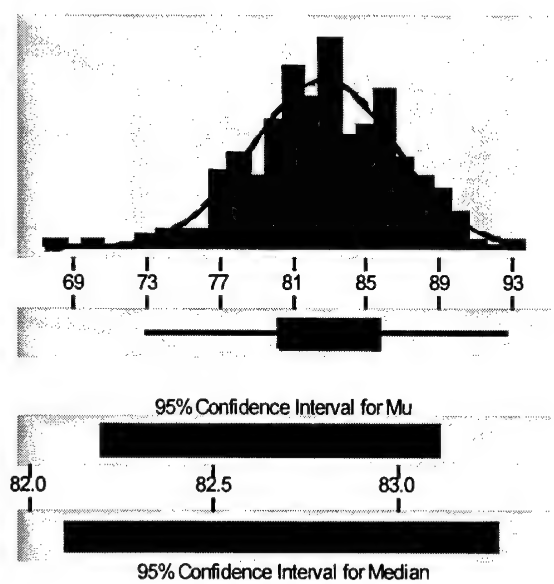
95% Confidence Interval for Sigma

3.8146 4.5031

95% Confidence Interval for Median

86.5301 87.7699

Descriptive Statistics



Variable: avg3067

Anderson-Darling Normality Test

A-Squared:	0.346
P-Value:	0.481
Mean	82.6530
StDev	3.8961
Variance	15.1799
Skewness	-3.4E-01
Kurtosis	0.421564
N	281
Minimum	67.6500
1st Quartile	80.2500
Median	82.7000
3rd Quartile	85.5750
Maximum	92.7000

95% Confidence Interval for Mu

82.1955 83.1105

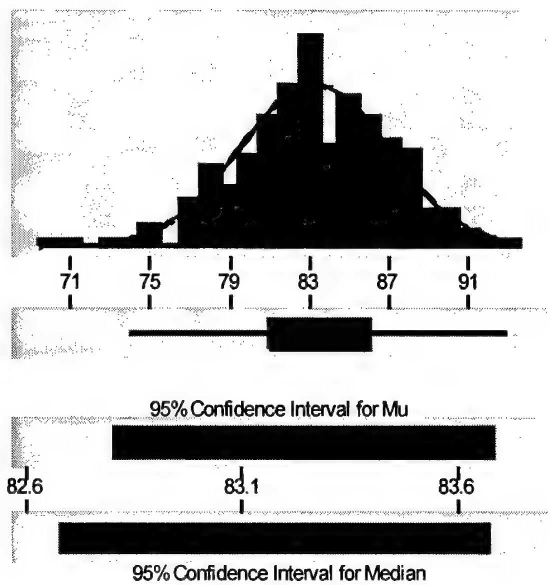
95% Confidence Interval for Sigma

3.5984 4.2480

95% Confidence Interval for Median

82.1000 83.2699

Descriptive Statistics



Variable: avg3090

Anderson-Darling Normality Test

A-Squared:	0.344
P-Value:	0.485
Mean	83.2454
StDev	3.7500
Variance	14.0625
Skewness	-3.0E-01
Kurtosis	0.253001
N	281
Minimum	70.0500
1st Quartile	81.0500
Median	83.2500
3rd Quartile	85.9250
Maximum	92.8500

95% Confidence Interval for Mu

82.8050 83.6857

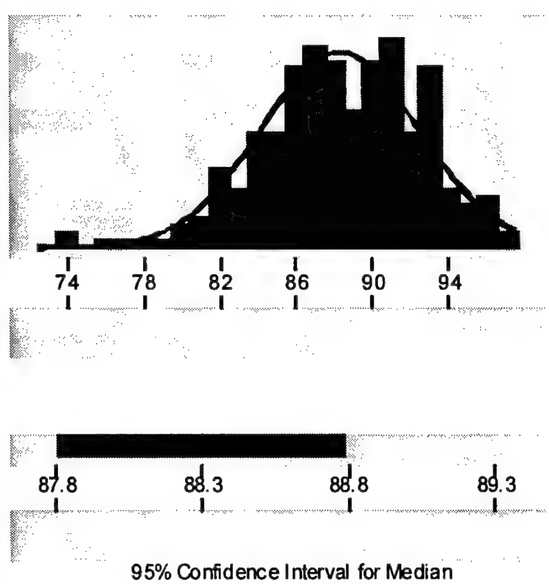
95% Confidence Interval for Sigma

3.4635 4.0886

95% Confidence Interval for Median

82.6801 83.6699

Descriptive Statistics



Variable: avg30112

nderson-Darling Normality Test

A-Squared:	0.590
P-Value:	0.123
Mean	88.2948
StDev	4.1375
Variance	17.1189
Skewness	-4.8E-01
Kurtosis	0.454137
N	281
Minimum	73.5000
1st Quartile	85.6500
Median	88.4000
3rd Quartile	91.1500
Maximum	96.8000

5% Confidence Interval for Mu

87.8090 88.7807

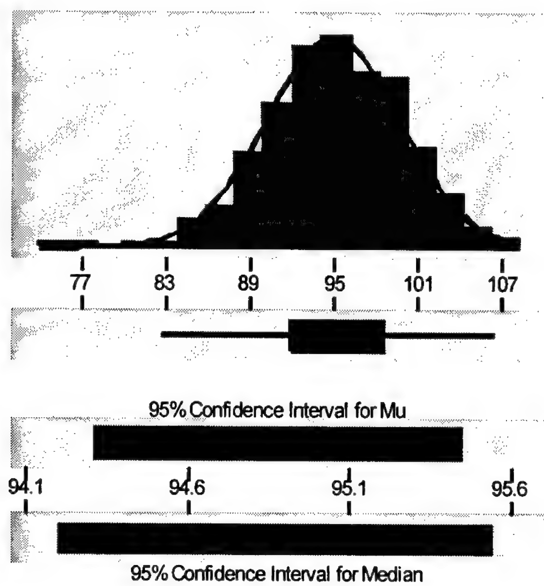
% Confidence Interval for Sigma

3.8214 4.5111

Confidence Interval for Median

87.8301 89.2899

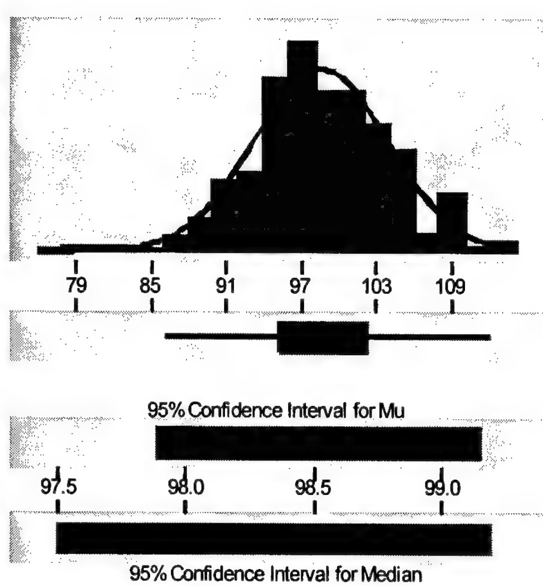
Descriptive Statistics



Variable: avg30135

Anderson-Darling Normality Test	
A-Squared:	0.429
P-Value:	0.307
Mean	94.8749
StDev	4.8366
Variance	23.3923
Skewness	-4.0E-01
Kurtosis	0.854189
N	281
Minimum	75.200
1st Quartile	91.950
Median	94.850
3rd Quartile	98.450
Maximum	106.400
95% Confidence Interval for Mu	
94.307	95.443
95% Confidence Interval for Sigma	
4.467	5.273
95% Confidence Interval for Median	
94.200	95.540

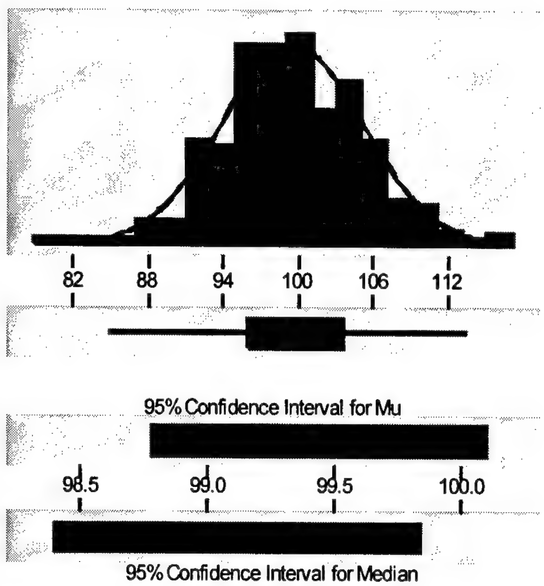
Descriptive Statistics



Variable: avg30157

Anderson-Darling Normality Test	
A-Squared:	0.404
P-Value:	0.352
Mean	98.5176
StDev	5.3646
Variance	28.7786
Skewness	-1.2E-01
Kurtosis	0.571480
N	281
Minimum	78.950
1st Quartile	95.200
Median	98.250
3rd Quartile	102.175
Maximum	112.850
95% Confidence Interval for Mu	
97.888	99.148
95% Confidence Interval for Sigma	
4.955	5.849
95% Confidence Interval for Median	
97.500	99.190

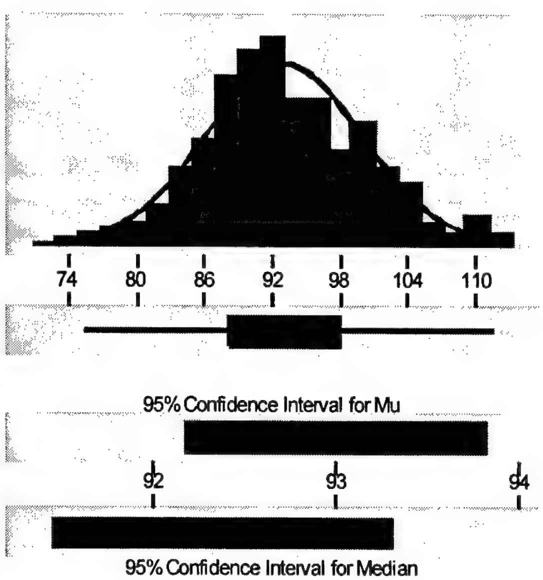
Descriptive Statistics



Variable: v30180

Anderson-Darling Normality Test	
A-Squared:	0.354
P-Value:	0.461
Mean	99.4388
StDev	5.6464
Variance	31.8820
Skewness	-2.7E-02
Kurtosis	0.416211
N	281
Minimum	79.500
1st Quartile	95.900
Median	99.200
3rd Quartile	103.400
Maximum	116.100
95% Confidence Interval for Mu	
98.776	100.102
95% Confidence Interval for Sigma	
5.215	6.156
95% Confidence Interval for Median	
98.400	99.840

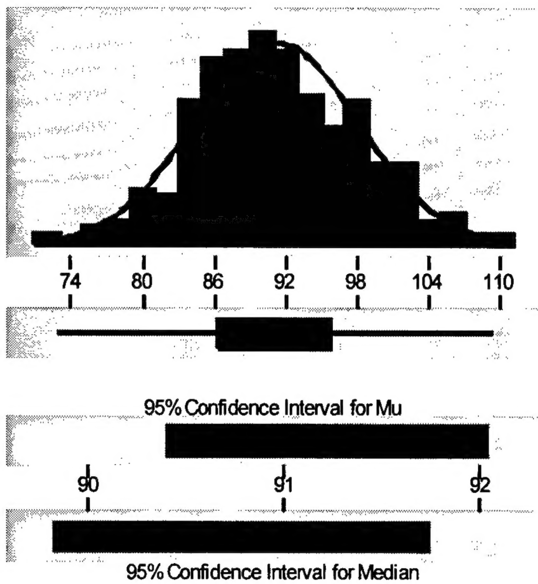
Descriptive Statistics



Variable: v600

Anderson-Darling Normality Test	
A-Squared:	0.643
P-Value:	0.093
Mean	93.0018
StDev	6.9529
Variance	48.3435
Skewness	0.168078
Kurtosis	-2.4E-02
N	281
Minimum	73.100
1st Quartile	88.150
Median	92.300
3rd Quartile	97.750
Maximum	111.600
95% Confidence Interval for Mu	
92.185	93.818
95% Confidence Interval for Sigma	
6.422	7.581
95% Confidence Interval for Median	
91.460	93.300

Descriptive Statistics



Variable: avg6022

Anderson-Darling Normality Test

A-Squared:	0.481
P-Value:	0.230
Mean	91.2235
StDev	6.9676
Variance	48.5476
Skewness	0.130512
Kurtosis	-1.3E-01
N	281
Minimum	71.450
1st Quartile	86.350
Median	90.800
3rd Quartile	95.800
Maximum	110.150

95% Confidence Interval for Mu

90.405 92.042

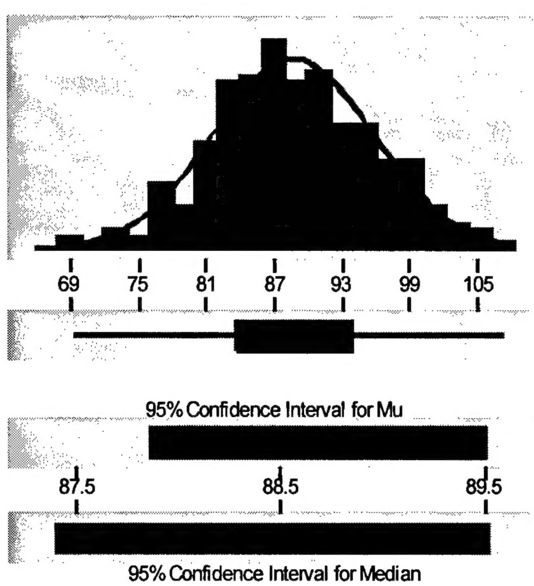
95% Confidence Interval for Sigma

6.435 7.597

95% Confidence Interval for Median

89.830 91.740

Descriptive Statistics



Variable: avg6045

Anderson-Darling Normality Test

A-Squared:	0.275
P-Value:	0.660
Mean	88.6833
StDev	7.0337
Variance	49.4724
Skewness	7.56E-03
Kurtosis	-1.7E-01
N	281
Minimum	68.550
1st Quartile	83.725
Median	88.600
3rd Quartile	93.700
Maximum	107.100

95% Confidence Interval for Mu

87.857 89.509

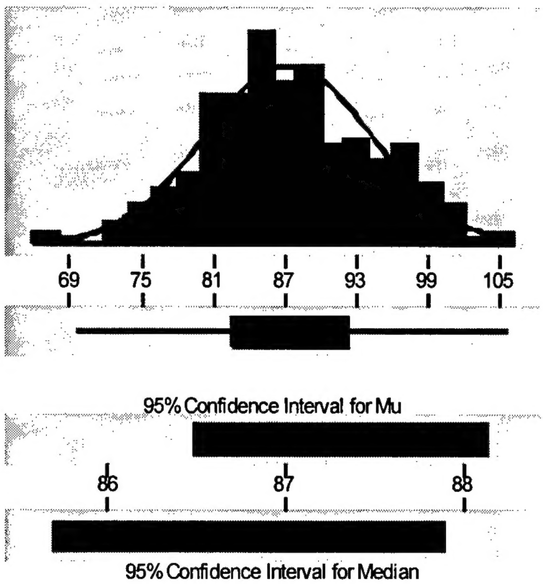
95% Confidence Interval for Sigma

6.496 7.669

95% Confidence Interval for Median

87.400 89.520

Descriptive Statistics



Variable: avg6067

Anderson-Darling Normality Test

A-Squared:	0.643
P-Value:	0.092
Mean	87.3100
StDev	7.0310
Variance	49.4346
Skewness	1.66E-02
Kurtosis	-1.2E-01
N	281
Minimum	66.100
1st Quartile	82.525
Median	86.850
3rd Quartile	92.250
Maximum	105.650

95% Confidence Interval for Mu

86.484 88.136

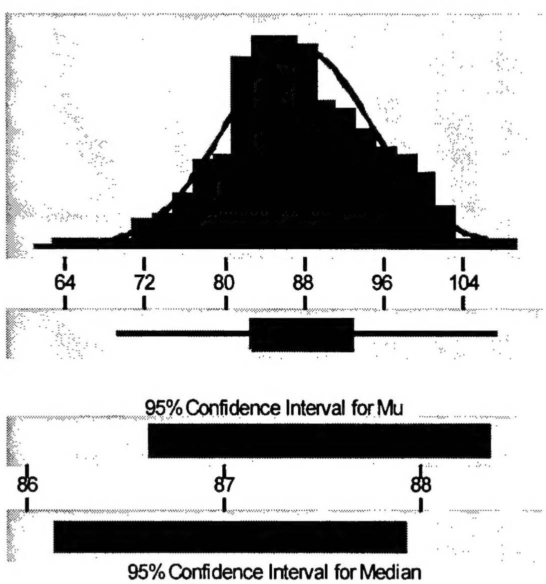
95% Confidence Interval for Sigma

6.494 7.666

95% Confidence Interval for Median

85.700 87.890

Descriptive Statistics



Variable: avg6090

Anderson-Darling Normality Test

A-Squared:	0.591
P-Value:	0.122
Mean	87.4835
StDev	7.3694
Variance	54.3085
Skewness	-1.4E-02
Kurtosis	-9.0E-02
N	281
Minimum	63.750
1st Quartile	82.725
Median	87.000
3rd Quartile	92.675
Maximum	107.250

95% Confidence Interval for Mu

86.618 88.349

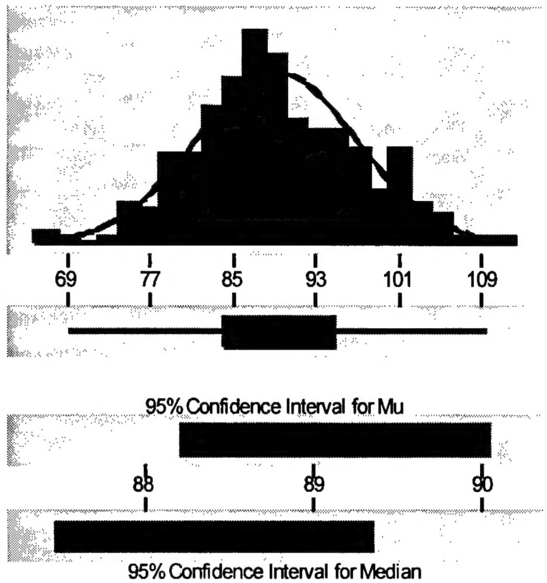
95% Confidence Interval for Sigma

6.806 8.035

95% Confidence Interval for Median

86.140 87.920

Descriptive Statistics



Variable: avg60112

Anderson-Darling Normality Test

A-Squared:	0.807
P-Value:	0.036
Mean	89.1313
StDev	7.7599
Variance	60.2168
Skewness	0.114019
Kurtosis	-1.4E-01
N	281
Minimum	66.850
1st Quartile	83.975
Median	88.300
3rd Quartile	94.550
Maximum	110.450

95% Confidence Interval for Mu

88.220 90.043

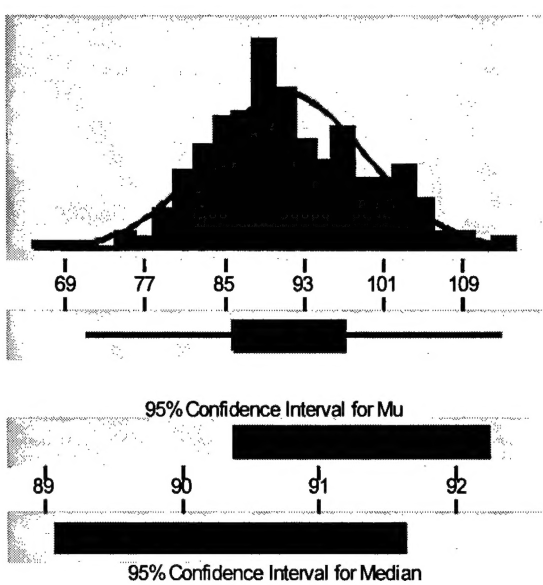
95% Confidence Interval for Sigma

7.167 8.461

95% Confidence Interval for Median

87.480 89.350

Descriptive Statistics



Variable: avg60135

Anderson-Darling Normality Test

A-Squared:	1.179
P-Value:	0.004
Mean	91.3085
StDev	7.9781
Variance	63.6509
Skewness	0.135905
Kurtosis	-3.4E-02
N	281
Minimum	66.100
1st Quartile	85.800
Median	90.100
3rd Quartile	96.850
Maximum	112.850

95% Confidence Interval for Mu

90.372 92.245

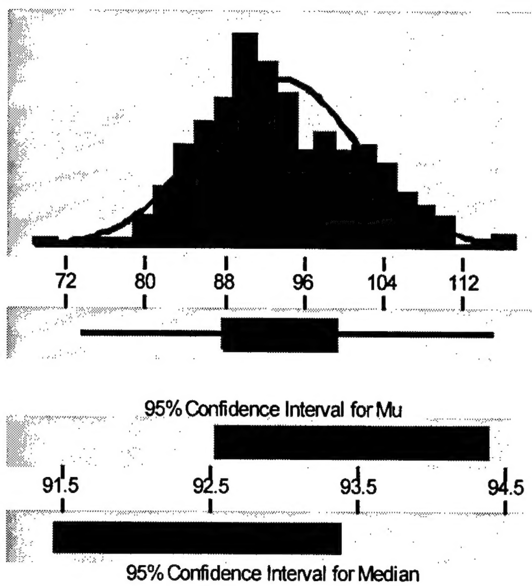
95% Confidence Interval for Sigma

7.369 8.699

95% Confidence Interval for Median

89.080 91.620

Descriptive Statistics



Variable: avg60157

Anderson-Darling Normality Test

A-Squared:	1.356
P-Value:	0.002
Mean	93.4552
StDev	7.8823
Variance	62.1308
Skewness	0.273481
Kurtosis	-1.8E-01
N	281
Minimum	70.400
1st Quartile	88.000
Median	92.300
3rd Quartile	99.100
Maximum	115.050

95% Confidence Interval for Mu

92.530	94.381

95% Confidence Interval for Sigma

7.280	8.594

95% Confidence Interval for Median

91.450	93.370